S E S ' 2 0 0 5

Scientific Conference "SPACE, ECOLOGY, SAFETY" with International Participation

10–13 June 2005, Varna, Bulgaria

A METHOD OF GENERALIZED ORTHOGONAL COMPLEMENTARY CODES APPLYING IN SPACE BASED RADARS

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Key words: Communication system signaling, Radars, Generalized Orthogonal Complementary Codes.

Abstract: *The experience, obtained in the local military conflicts during the past twenty years, shows persuasively that the radioelectronic war has great importance for the combat success. Due to this reason, the radiocommunication systems, including radars, are high priority targets. After finding of their location they are attacked immediately. For their destroying special intelligent rockets are designed. With regard at present, the measures, providing high level of secrecy of the radiocommunication systems, are studied intensively.*

A promising method for increasing the radiocommunication and radar system secrecy is the applying of complex wideband radiosignals. They provide large performance range and high distance resolution. Simultaneously, due to the small spectral density of these signals, the discovering of radiosystem is very hard.

With regard to the positive features of the complex wideband radiosignals, our paper is focused on a particular class of these signals, named generalized orthogonal complementary codes (GOCCs). Namely, we study a method for GOCC applying in the spacecraft based radars.

INTRODUCTION

The experience, obtained in the local military conflicts during the past twenty years, shows persuasively that the radioelectronic war has great importance for the combat success. Due to this reason, the radiocommunication systems, including radars, are high priority targets. After finding of their location they are attacked immediately. For their destroying special intelligent rockets are designed. This situation shows that providing high level of secrecy has great importance for some military applications [1], especially for the radars, including these based on spacecrafts.

As known, military radars comprise two subsystems, named primary and secondary radar respectively. The primary radar is used for discovering of targets, measurement of their coordinates and for following of their trajectories ("tracking"). The most primary radars work as "active-passive" systems. The active operation is the sending by radar transmitter series of powerful electromagnetic pulses. The passive operation is scattering by targets the small part of power of the sent pulses, which returns to the radar receiver. The secondary radars are assigned for solving of a particular but very important military problem – dividing the observed objects in two main classes, named "friend" and "foe". Due to the significance of the target identification, the secondary radars are "active-active" systems. It means that our (friend) targets have a transmitter, which is activated by the pulses, sent by observing radars. As a result the targets return the so-named "responses", which are messages, containing special coded data. This data is verified in the secondary radar receivers and if it matches to a "pattern", it is decided that target is "friend".

Otherwise, the decision is “foe”. Despite of great importance of “friend or foe” identification (IFF), the responses of our objects could disclose their positions. Consequently, the providing high level of secrecy of the secondary radars has a crucial role on the battle field.

A promising method for increasing the radiocommunication and radar system secrecy is the applying of complex wideband radiosignals. They provide large performance range and high distance resolution. Simultaneously, due to the small spectral density of these signals, the discovering of radiosystem is very hard. With regard at present, the measures, providing high level of secrecy of the radiocommunication systems, are studied intensively.

At present a great number of complex wideband radiosignals are proposed and used in practice. Nevertheless, it ought to emphasize that the employment so-named multicarrier direct-sequence code-division multiple access (DS-CDMA) [2] leads to significant improvement of radiocommunication system secrecy. In the DS-CDMA approach each of the carrier frequencies in a multicarrier system is multiplied by a spreading sequence unique to each user. This technique in a comparison with all other wideband approaches demonstrates a number of desirable features, including narrow-band interference suppression and a lower required chip rate than that of a single-carrier system occupying the same total bandwidth. The lower required chip rate is a result of the fact that the entire bandwidth is divided equally among frequency bands. This also allows the receiver to incorporate parallelized signal processing, with each of the parallel branches having a much lower computational load than that of a single serial processor for a single-carrier system occupying the same bandwidth. In addition, it is easier to implement the parallel receiver architecture of a number of carriers than a larger order RAKE [2].

With regard to the positive features of the multicarrier DS-CDMA utilizes complex wideband radiosignals, our paper is focused on a particular class of these signals, named generalized orthogonal complementary codes (GOCCs). Namely, we study a method for GOCCs applying in the spacecraft based radars.

Our paper is organized as follows. First, the basics of GOCCs are recalled. After that an algebraic method for synthesis of GOCCs is proved. Finally, the advantages and possible areas of application of our method in the spacecraft based radars are discussed.

A METHOD OF GENERALIZED ORTHOGONAL COMPLEMENTARY CODES APPLYING IN SPACE BASED RADARS

As mentioned above, the secondary radars form a dynamic multiuser radiocommunication system which must possess simultaneously significant level of secrecy, high rate of information transfer and great reliability. Satisfying of these requirements is a hard technical problem. By our opinion it could be solved by applying of GOCCs in the secondary radars. With regard the basics of GOCCs will be given in the beginning of this section of our paper.

The main idea of the GOCC is assigning a unique set (family) of distinct spreading sequences to each user in a multicarrier DS-CDMA system [2], [3]. So the GOCC can be viewed as a set of ***N*** *families, where every family contains* ***M*** *sequences and every sequence comprises* ***n*** *complex number with absolute value* ***1****. The elements of the sequences describe mathematically the phase manipulation of elementary phase pulses (chips). The families of sequences have the unique ability to eliminate the multiple-access interference (MAI) in asynchronous multicarrier DS-CDMA systems. The reduction in MAI minimizes the effect of the near–far problem, as well as other MAI-induced errors. Therefore, the proposed system can support more users for a fixed-error probability constraint. Furthermore, the autocorrelation sidelobes are canceled. As a result information symbols may be packet more closely together, which increases the data rate* *achievable by a single user [2], [3]. The proposed system does have some disadvantages. One disadvantage is that the system is not as resistant to frequency-selective fading, as all frequency components are important for effective reduction of autocorrelation sidelobes, and it is this reduction that allows for individual users to signal at a higher data rate. However, even with this disadvantage, the system appears well suited to certain types of communication channels, such as stable phase-coherent channels, or Rician channels with a strong line-of-sight (LOS) path, where the effects of frequency-selective fading are minimal.*

In a DS-CDMA system it is desirable the autocorrelation of spreading sequences to be zero for all nonzero shifts. While it is not possible to construct a single binary sequence of values $\{-1,+1\}$ having an (aperiodic) autocorrelation function equal to zero for all nonzero shifts, it is possible to construct two such sequences whose autocorrelation functions, when coherently added, result in a function having value zero for nonzero shifts. An example of such a pair of sequences is the Golay sequences [4]. With regard we shall recall the following definition.

***Definition**: An orthogonal complementary code (OCC) is a set of **N** families, where every family comprises **M** sequences $\{\zeta_k(j)\}_{j=0}^{n}$, $k=1,2,...,M$ so that:*

- the aggregated aperiodic autocorrelation function (ACF) of all sequences in an arbitrary family has only main lobe without any side lobes (i.e. it exhibits the so-named "thumbtack" shape);

- the aperiodic aggregated cross-correlation function (CCF) between corresponding sequences of any two distinct families remains low throughout.

Often it is said that the families of an OCC are "mutually orthogonal" (MO).

MAI in a DS-CDMA system mainly results from nonzero cross correlation between the intended user's spreading sequence and an unintended user's spreading sequence when matched filtering is used. In fact, for maximal connected sets of m-sequences or Gold sequences of length $L=2^n-1$, the peak cross correlation magnitude is $1+2^{\lfloor(n+2)/2\rfloor}$. This too points to the possibility of reducing MAI using a set of multiple spreading sequences per user in the multicarrier DS-CDMA system.

At present two methods for synthesis of orthogonal complementary codes (OCCs) are known. First of them utilizes an arbitrary Hadamard matrix in order to create the families of an OCC. The second one, introduced by Tseng and Liu, is based on complementary pairs, invented by Golay [4], [5].

An important shortcoming of the classical approaches to design of the OCCs is usage of only binary manipulation, which restricts the information rate transfer and the flexibility of the radiocommunication system. With regard we shall generalize the classical Tseng and Liu's method for synthesis of OCCs [5] so that arbitrary phase modulation to be possible. Our generalization is based on the following theorems.

***Theorem 1:** Let **A** be a vector-column, which entries are **M** complementary sequences, i.e.:*

$$\mathbf{A}^T=\left\{\{\zeta_1(j)\}_{j=0}^{n-1},\{\zeta_2(j)\}_{j=0}^{n-1},....,\{\zeta_M(j)\}_{j=0}^{n-1}\right\} \quad (1)$$

*where the upper index "^{T}" means "transposition" and **M** is an even number. Then the vector-columns **A** and **B**, where **B** is defined by:*

$$\mathbf{B}^T=\left\{\{\zeta_2^*(j)\}_{j=n-1}^{0},\{-\zeta_1^*(j)\}_{j=n-1}^{0},....,\{\zeta_M^*(j)\}_{j=n-1}^{0},\{-\zeta_{M-1}^*(j)\}_{j=n-1}^{0}\right\}, \quad (2)$$

form a GOCC with $N=2$ families. Here the symbols "" and "$\}_{j=n-1}^{0}$" in (2) mean "complex conjugation" and "reverting of the order of the sequence".*

Proof: *In contrast with classical method for synthesis of OCCs, where only binary phase modulation (or binary phase shift keying (BPSK)) is applied, we shall examine the common case. This means that arbitrary **m**-ary ($m \geq 2$) phase manipulation (or MPSK) is possible and consequently the elements of the sequences of an GOCC are **m**-th roots of the unity:*

$$\forall \zeta_k(j) \in \left\{ exp\left(i\frac{2\pi l}{m} \right);\ l = 0, 1, ..., m-1 \right\};\quad i = \sqrt{-1}. \tag{3}$$

*This generalization makes the Tseng and Liu's method inapplicable, because it deals with odd (i.e. $(+1,-1)$; $(-1,+1)$) and even (i.e. $(-1,-1)$; $(+1,+1)$) pairs. Due to this reason we shall use of the so-named "method of formal polynomials (or formal power sums of a single variable)" [6]. According to this method, the values of the ACF of any sequence of the matrix **B** are the coefficients of the polynomial product:*

$$P_{kk}(x) = \tilde{F}_k(x).\tilde{F}_k^*(x^{-1}),\ k = 1,2,...,M\,, \tag{4}$$

where:

$$\tilde{F}_k(x) = \left[\pm \zeta_k(0).x^{n-1} \pm \zeta_k(1).x^{n-2} \pm ... \pm \zeta_k(n-3).x^2 \pm \zeta_k(n-2).x \pm \zeta_k(n-1) \right], \tag{9}$$

$$\tilde{F}_k^*(x) = \left[\pm \zeta_k^*(0).x^{-(n-1)} \pm \zeta_k^*(1).x^{-(n-2)} \pm ... \pm \zeta_k^*(n-2).x^{-1} \pm \zeta_k^*(n-1) \right]. \tag{10}$$

The polynomial (9) can be transformed as follows:

$$\begin{aligned} \tilde{F}_k(x) &= \left[\pm \zeta_k(0).x^{n-1} \pm ... \pm \zeta_k(n-2).x \pm \zeta_k(n-1) \right] = \\ &= x^{n-1}\left[\pm \zeta_k(n-1).x^{-(n-1)} \pm ... \pm \zeta_k(1).x^{-1} \pm \zeta_k(0) \right] = x^{n-1}.\left[\pm F_k(x^{-1}) \right] \end{aligned} \tag{11}$$

Here:

$$F_k(x) = \left[\zeta_k(n-1).x^{n-1} + \zeta_k(n-2).x^{n-2} + ... + \zeta_k(2).x^2 + \zeta_k(1).x + \zeta_k(0) \right]. \tag{12}$$

It can be analogously shown that:

$$\tilde{F}_k^*(x^{-1}) = x^{-(n-1)}\left[\pm \zeta_k^*(n-1).x^{n-1} \pm ... \pm \zeta_k^*(1).x \pm \zeta_k^*(0) \right] = x^{-(n-1)}.\left[\pm F_k^*(x) \right] \tag{13}$$

From (11) and (13) it is straightforward that:

$$P_{kk}(x) = \tilde{F}_k(x).\tilde{F}_k^*(x^{-1}) = F_k(x^{-1}).F_k^*(x) = \left[F_k(x).F_k^*(x^{-1}) \right]^*. \tag{14}$$

*Consequently, the aggregated ACF of the sequences of the vector-column **B** has values which are coefficients of the sum:*

$$P_{\mathbf{B}}(x) = \sum_{k=1}^{M} P_{kk}(x) = \left[\sum_{k=1}^{M} F_k(x).F_k^*(x^{-1}) \right]^*. \tag{15}$$

*Now it should be noted that the aggregated ACF of the sequences of the vector-column **A** has values which are coefficients of the sum*

$$P_{\mathbf{A}}(x) = \sum_{k=1}^{M} F_k(x).F_k^*(x^{-1}). \tag{16}$$

*According to complementary property of the sequences of the vector-column **A** their aggregated ACF has thumbtack shape (i.e. it has only a main lobe and does not have any side lobes), which means that:*

$$P_{\boldsymbol{A}}(x) = Mn\,. \tag{17}$$

Consequently:

$$P_{\boldsymbol{B}}(x) = [Mn]^{*} = Mn\,, \tag{18}$$

*which shows that the sequences of the vector-column **B** possess the complementary property also.*

*Now, in order to prove Theorem 1 it is necessary to demonstrate that aggregated CCF of the corresponding sequences of the vector-columns **A** and **B** is zero everywhere.*

*According to method of formal polynomials, the values of the aggregated CCF of the corresponding sequences of the vector-columns **A** and **B** are the coefficients of the polynomial product:*

$$\begin{aligned} P_{\boldsymbol{AB}}(x) &= \sum_{k=1}^{M/2}\left\{F_{2k-1}(x).\left[\widetilde{F}^{*}_{2k}(x^{-1})\right]^{*} + F_{2k}(x).\left[-\widetilde{F}^{*}_{2k-1}(x^{-1})\right]^{*}\right\} = \\ &= \sum_{k=1}^{M/2}\left[F_{2k-1}(x).\widetilde{F}_{2k}(x^{-1}) - F_{2k}(x).\widetilde{F}_{2k-1}(x^{-1})\right] \end{aligned}, \tag{19}$$

From (11) it follows that:

$$F_{2k}(x).\widetilde{F}_{2k-1}(x^{-1}) = x^{n}.\widetilde{F}_{2k}(x^{-1}).\widetilde{F}_{2k-1}(x^{-1}) = F_{2k-1}(x).\widetilde{F}_{2k}(x^{-1})\,. \tag{20}$$

The accounting of (20) in (19) shows:

$$P_{\boldsymbol{AB}}(x) = 0\,, \tag{21}$$

*which proves that aggregated CCF of the corresponding sequences of the vector-columns **A** and **B** is zero everywhere.*

*In order to finish the proof of Theorem 1 it is enough to see that the $M \times 2$ matrix **C**:*

$$\boldsymbol{C} = [\boldsymbol{A}\ \ \boldsymbol{B}] \tag{22}$$

*which first and second columns are the vectors **A** and **B** respectively is a GOCC, containing $N = 2$ families.*

The importance of Theorem 1 follows from the fact that it gives an algorithm for creating of an "initial" GOCC with $N = 2$ families. This algorithm is very effective because at present several methods for synthesis of families of complementary sequences are known.

*Now we shall state the following Theorem 2 which gives a general algorithm for building of a new GOCC with **2N** families if a GOCC with **N** families is known.*

***Theorem 2:** Let **C** be a $M \times N$ matrix so that:*

*- its columns CC_s contain the sequences with equal length **n**:*

$$CC_s = \left\{\{\zeta_{1s}(j)\}_{j=0}^{n-1}, \{\zeta_{2s}(j)\}_{j=0}^{n-1}, \ldots, \{\zeta_{Ms}(j)\}_{j=0}^{n-1}\right\},\ s = 0,1,\ldots,N-1\,; \tag{23}$$

- its columns CC_s are families of MO sequences.

Then the derivative matrix:

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{C}\otimes\boldsymbol{C} & -\boldsymbol{C}\otimes\boldsymbol{C} \\ -\boldsymbol{C}\otimes\boldsymbol{C} & \boldsymbol{C}\otimes\boldsymbol{C} \end{bmatrix}, \tag{24}$$

*is a GOCC with **2N** families and every family has **2M** sequences. Here the symbol "$\otimes$" means "interleaving of the elements of every column of the matrix C":*

$$\begin{aligned}&\{\zeta_{rs}(0),\zeta_{rs}(0),\, \zeta_{rs}(1),\zeta_{rs}(1),\, ...\zeta_{rs}(n-1),\zeta_{rs}(n-1)\},\\ &r=0,1,,...,M-1,\ s=0,1,,...,N-1.\end{aligned} \quad (25)$$

The proof will be omitted because it can be done by arguments, analogous to these, used above.

CONCLUSIONS

In this paper, we generalize the classical technique for multicarrier DS-CDMA that employs a set of spreading sequences for each user. The phase manipulation of the sequences can be arbitrary in contrast with classical approach which utilizes only binary manipulation. This result is obtained by proving a general method for GOCC synthesis. It consists of two steps:

- creating "initial" GOCC with $N=2$ families from a known family of complementary sequences;

*- building of a new GOCC with **2N** families from a base GOCC with **N** families.*

Our method could be very useful in the design of the modern secondary radars because the usage of the GOCCs enables to eliminate MAI caused by simultaneous responses of our objects on the battle field. Moreover the data rate and capacity of all radar system can be significantly increased.

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