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BAROCLINICITY AND CONSEQUENCE CONDITIONS FOR VORTICITY FORMATIONS IN ACCRETION DISCS

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Abstract

Due to arising of instabilities in accretion discs, turbulence and vortices may formed. The investigations of these formations are important, because of their role for angular momentum transport in accretion discs. Here we present our survey of specific type of instability – baroclinic instability. We explain the possible reasons and mechanisms for its appearance in accretion discs. Using the appropriate equations we find the conditions for the vorticity formations, as a result of baroclinic instability work.

Introduction

It is shown by analytical consideration and three-dimensional hydro simulations (Klahr & Bodenheimer 2004), that in discs, particularly in protoplanetary discs, have a negative entropy gradient, which makes them baroclinic. Two-dimensional numerical simulation gives the result for an unstable baroclinic flow and for producing turbulence.

In barotropic simulations hydrodynamical turbulence usually rapidly damps and in principle the barotropic flow is stable and does not engender turbulence.

Baroclinic instability arises in rotating fluids when the surfaces of constant pressure and the constant density do not re-cover, there is an angle between them. Rotating centrifugally supported systems can be hydrodynamically unstable to non-axisymmetric perturbation under the influence of radial entropy gradient. Even in the cases when the flow is stable to other criterions, the radial entropy gradient produces Baroclinic instability and oscillates nonlinear waves.

The existence of baroclinicity requires that the accretion flow has to be non-isothermal. Than the disc have an entropy gradient and instability may arised.

The models of rotating stars show the deviation of initially rotation, which allows in their structure any rate of baroclinicity. It is supposed that turbulent motions as a source of viscous pressure are driven by instabilities inherent to baroclinic systems.

The turbulent motions are in the connections with instabilities, existing in baroclinic systems (Cabot 1984). Such baroclinic instability is considered to be one possible mechanism to evolving of turbulence in a disc. That turbulence creates pressure waves, Rossby waves and especially vortices in the discs. Also, the nonlinear evolution of this instability may lead to formation of great-scale vortically structure in the disc.

Particularly, the non-axisymetric baroclinic instability received your energy of temperature variation over isobars by exchange of fluid elements near isobars and isotherms. At such non-axisymetrical motions the angular momentum doesn't conserve and it transfers by the perturbation pressure.

We represent in the paper these processes in the case of accretion discs, which are the hydrodynamic flow around compact object in binary star systems.

I Conditions for baroclinicity and vorticity formations

1. General conditions.

A barotropic shear flow is in principle stable and does not develop turbulence. In barotropic case, hydrodynamical turbulence usually rapidly decays and is not able to sustain itself. How we can see this from vortical equation. Using the presentation of Lovelace (Lovelace at all. 1999) for this equation we show it now in this general view:

(1) $\frac{D}{Dt}\left(\frac{\Psi_z}{\rho}\right) = \frac{\nabla\rho \times \nabla P}{\rho^3}$, where $\Psi_z = \hat{z}.\nabla \times v$ is the vorticity.

Where $\frac{D}{Dt} = \frac{\partial}{\partial t} + v.\nabla$, than the above expression takes a form:

(2) $\left(\frac{\partial \Psi}{\partial t} + v.\nabla\right)\frac{1}{\rho} = \frac{\nabla\rho \times \nabla P}{\rho^3}$

For a barotropic flow, the right side of this expression is zero and each element conserves its specific vorticity. This cannot lead to forming new vortices.

In the other case of the non-barotropic flow the term $\nabla\rho \times \nabla P \propto \nabla T \times \nabla S$ ($S = P/\rho^{\Gamma}$, S is the entropy of the disc matter) destroys this conservation and the pressure force may generate vortices in the flow. This flow proves to be baroclinic.

2. The dependence of entropy variation in baroclinic discs and the influence of radial entropy gradient for the existence of baroclinicity.

How we may explain that dependence. We start with energy balance equation for viscous, no ideal fluid:

(3) $\frac{\partial}{\partial t}\left[\rho\left(\frac{1}{2}v^2 + \varepsilon + \Phi\right)\right] + \nabla.\left[\rho v\left(\frac{1}{2}v^2 + h + \Phi\right)\right] = \rho T\frac{dS}{dt}$

Where:
$\frac{\partial}{\partial t}\left[\rho\left(\frac{1}{2}v^2+\varepsilon+\Phi\right)\right]$ - total energy density, where the first term denote kinetic energy, the second is an internal energy and the third is the gravitational field.

$\left[\rho v\left(\frac{1}{2}v^2+h+\Phi\right)\right]$ - total energy flux, where $h=\varepsilon+P/\rho$ is the enthalpy.

Where for a viscous fluid with negligible heat flow, the rate of increase of entropy due to viscous dissipation is:

(4) $\rho T\frac{dS}{dt}=\varsigma\nabla^2.v^2+2\eta\sigma$

where ς is the bulk viscosity; η is the shear viscosity / $\eta/\rho=\nu$ /; $\sigma=d\Sigma/dt$ - the shear tensor;

Further, we need to express the conservation equation of entropy. We can combine the rate of viscous dissipation with equation of mass conservation to obtain the following:

(5) $\frac{\partial(\rho S)}{\partial t}+\nabla.(\rho S v)=\frac{\varsigma\nabla^2.v^2+2\eta\sigma}{T}$

- where the first term of left hand side express the rate of change of entropy density; the second term is the divergence of entropy flux;
- the term of the right hand side denotes the rate of production of entropy;

The temperature evolution is given with the expression:

(6) $\frac{\partial T}{\partial t}+v.\nabla T=0$

We need to express the change of the radial entropy gradient and its direction of distribution. Using the relations of Klahr & Bodenheimer (2000) we may write down the next relation:

(7) $\frac{dS}{dr}=c_v\frac{d}{dr}\log P\rho^{-\gamma}$,

where c_v is the specific heat and γ is the adiabatic index.

We are studying the case of volume density ρ. Then we varying the values of the adiabatic index, corresponding to the accretion flow matters and take note to the system of equations (3-7). Performing an analytical consideration the obtaining result show that the radial entropy gradient is negative and in this reason the baroclinicity in the flow exists. It is seen, also, from Figure 1:

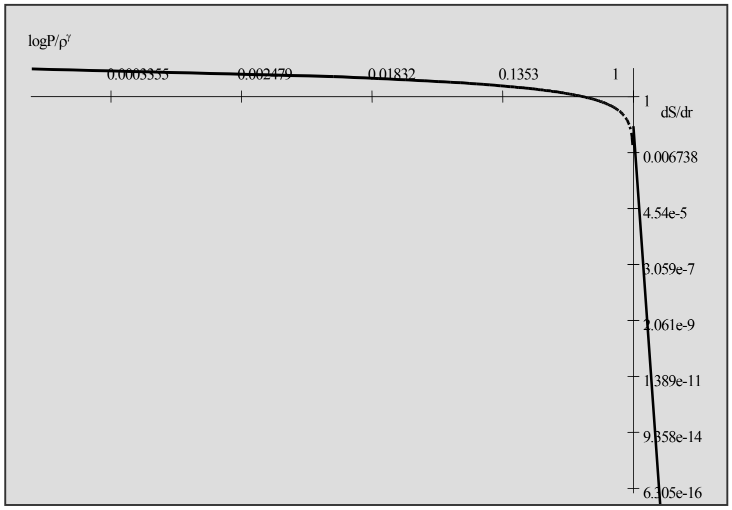

Fig.1. Radial entropy gradient distribution / $\gamma = (1.3 \div 1.43)$ **/.**

II. A dependence from right hand side of vortical equation, applied for the accretion discs

We use the Navier-Stokes equation and obtained vortical transport equation, which in general case is:

(8) $$\frac{\partial v}{\partial t} + v.\nabla v = -\frac{1}{\rho}\nabla P + \nu\nabla^2 v + F$$

Following the baroclinicity conditions, we turn them to our case and receive the next situation:
The accretion flow consists of rotating fluid and we write down the appropriate form of the Navier-Stokes equation:

(9) $$\frac{\partial v}{\partial t} + v.\nabla v = -\frac{1}{\rho}\nabla P - \Omega\times(\Omega\times r) - 2\Omega\times v + \nu\nabla^2 v$$

where:

ρ - is the mass density of the flow; we consider $\rho \neq const$ than the radial entropy gradient exist;
v - is the velocity of the flow;
P - is the pressure;
ν - is the kinematic viscousity;
$\Omega\times(\Omega\times r)$ - is the centrifugal force of the rotating accretion flow;
$2\Omega\times v$ - express the Coriolis force;

Here, we use again a curl of this expression and we obtain the next form of the vortical transport equation:

(10) $$\frac{\partial \Psi}{\partial t} = -\Psi(\nabla.v) - (v.\nabla)\Psi - \frac{\nabla p \times \nabla \rho}{\rho^2} + D\nabla^2\Psi$$

We rewrite this equation in the form:

(11) $$\frac{\partial \Psi}{\partial t} + \Psi(\nabla.v) + (v.\nabla)\Psi = -\frac{\nabla p \times \nabla \rho}{\rho^2} + D\nabla^2\Psi$$

where: Ψ - is the vorticity
D - is the diffusion coefficient (or matrix of the transport coefficient);

We consider non-ideal, viscous, compressible fluid and we cannot neglect the right hand side of this equation. Following the baroclinicity conditions we need right hand side of this equation to be different of zero. This result means that the vorticity formations may generate in our baroclinic accretion flow.

The analytical consideration shows that the diffusion coefficient in the expression (11) is not a constant and we may write D over r and φ direction:

$$\frac{\partial \Psi}{\partial t} + \Psi_r(\nabla.v_r) - (v_\varphi.\nabla)\Psi_\varphi = -\frac{\nabla p \times \nabla \rho}{\rho^2} + \frac{D_r}{D_\varphi}\nabla^2\Psi$$

These findings will not change the situation, the vorticity generation increased still further. Thereby, the second condition for the baroclinicity and relevant instability is fulfilled and we may assert that they operate as well in the accretion discs.

Using again the relations and terms in Navier-Stokes equation and vortical transport equation we reach to the next conditions:

The baroclinicity of the general flow is given by the baroclinic term (Klahr 2004):

$\nabla\rho(r,z,\varphi)\times\nabla p(r,z,\varphi) \neq 0$

The important for the instability is that non-axisymetric deviations from the mean state can lead to the rise of the baroclinic term even in two dimensions:

$\nabla\rho(r,\varphi)\times\nabla p(r,\varphi) \neq 0$

and vorticity can be generated.

In other words, the instability can arise if there is an inclination between the density and pressure gradients.

After analytical consideration and laboratory experiments, it is obtained the visual simulation of how does the baroclinic instability development. It is shown on the figure below. There are three frames in which we see different stages of baroclinic instability. In the third frame the instability is in its final stage and it is observe the turbulization of region over radial direction.

Fig. 2. Developing of baroclinic instability in radial direction.

Conclusion

The main statement for the baroclinicity, in this paper, is the negative radial entropy gradient. Thus also the pressure and density gradients do not point in the same direction. It is found that the global baroclinic instability produces turbulence in discs, which is of importance for angular moment transport in the flow, respectively for the existence of accretion discs.

Our considerations are based on the hydrodynamical equations and their analytical simulations, applied to the accretion discs essence and properties.

The baroclinic instability has been studied in the area of meteorology and oceanography. There are theoretical models and laboratory experiments, which may help us to understand this instability more basically.

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