S E N S ' 2 0 0 6

Second Scientific Conference with International Participation

SPACE, ECOLOGY, NANOTECHNOLOGY, SAFETY

14 – 16 June 2006, Varna, Bulgaria

PHYSICS OF LIGHT-INDUCED FORCES: CLASSICAL EFFECT AND INTERACTIONS DUE TO FLUCTUATING ELECTROMAGNETIC FIELD

Z. D. Genchev

Institute of Electronics, Bulgarian Academy of Sciences, zgenchev@ie.bas.bg

Key words: optical forces, van der Waals interactions, theory of perturbation

Abstract

We report the general theory to compute the force due to light upon a particle. After that we focus our attention to Van der Waals forces in plane parallel geometry for an one-dimensional crystal with an homogeneous inclusion embedded in its volume.

1. Introduction

The demonstration of mechanical action on small particles with radiation pressure was done by Ashkin [1]. In the past several theoretical works on optical forces used approximations by splitting the force into three parts: the gradient, scattering and absorbing forces [2]. However, a rigorous calculation requires the use of the Maxwell's stress tensor. The relation between the Maxwell's stress tensor and the force $\vec{F}$ on an object illuminated by an electromagnetic (e.m.) wave with angular frequency ω is given by the following equation

$$\vec{F} = \mathrm{Re}[\int_S \{[\vec{E}(\vec{r},\omega).\vec{n}]\vec{E}^*(\vec{r},\omega) + [\vec{H}(\vec{r},\omega).\vec{n}]\vec{H}^*(\vec{r},\omega) - \frac{1}{2}[\left|\vec{E}(\vec{r},\omega)\right|^2 + \left|\vec{H}(\vec{r},\omega)\right|^2]\vec{n}\}d\vec{r}] \qquad (1.1)$$

Here S is the surface enclosing the object, $\vec{n}$ is the local outward unit normal, the asterisk denotes the complex congugate, and Re represents the real part of a complex number. This equation is written in CGS units for an object in vacuum. If the object (the particle) is a sphere illuminated by a plane wave with an incident wave vector $\vec{k}_0 (k_0 = \left|\vec{k}_0\right| = \frac{\omega}{c})$ the force is given by the Mie scattering result [3]

$$\vec{F}_{Mie} = \frac{1}{8\pi}\left|\vec{E}_0\right|^2 (C_{ext} - <\cos\theta> C_{sca})\frac{\vec{k}_0}{k_0} \qquad (1.2)$$

where Cext denotes the extinction cross section, Csca the scattering cross section and <cosθ> the average of the cosine of scattering angle.In this paper we concentrate our attention to the case of forces induced by a fluctuating e.m. field (no incident plane wave as in (1.2)). These forces are due to thermal fluctuations; They exist even at zero temperature (T=0) due to quantum effects and are known as Van der Waals (VdW) or Casimir dispersion forces [4]. In the VdW limit the retardation of the e.m. interactions can be neglected, and in the Casimir limit the effect of retardation is considerable. We will report some extensions of the rigorous continuum model developed in [5] by Lifshitz and others who derived an expression for the e.m. fluctuational interaction between two macroscopic semiinfinite bodies separeated by a plane-parallel slab of finite thickness.

2.Formulation of the problem. The Green's dyadic function in an one dimensional inhomogeneous medium

Our purpose here is to derive two scalar differential equations and boundary conditions to them. We will show that these two scalar equations determine the whole spatial information for the Green's dyadic function $D_{lk}(\vec{r},\vec{r}^{'};\xi_n)$. According to general theory [6] the tensor D_{lk} is needed for calculation of the pressure (the force per unit area) acting in a medium having z coordinate dependent dielectric permittivity $\varepsilon\ (\omega,z)$. The starting point is the equation for $D_{lk}(\vec{r},\vec{r}^{'};\xi_n)$

$$\left\{\varepsilon(z;i\xi_n)\frac{\xi_n}{c^2}\delta_{jl}+\frac{\partial^2}{\partial x_j\partial x_l}-\Delta_3\delta_{jl}\right\}D_{lk}(\vec{r},\vec{r}^{'};\xi_n)=-4\pi\delta(\vec{r}-\vec{r}^{'})\delta_{jk} \qquad (2.1)$$

where Δ_3 is the 3D Laplace operator, $x_1=x,x_2=y,x_3=z$. Here $\varepsilon(z)\equiv\varepsilon(z;i\xi_n)$,:

$$\xi_n=n\frac{2\pi kT}{\hbar},n=0,1,2,... \qquad (2.2)$$

where, k is the Boltzmann constant, T is the temperature, j=1,2,3, l=1,2,3. Let $\bar{D}_{lk}(\vec{r},\vec{r}^{'};\xi_n)$ be the solution to (2.1) for a hypothetical homogeneous medium having for all z a scalar permittivity $\varepsilon(\omega,z_0)$ Then we introduce the notations

$$D^{'}_{lk}(\vec{r},\vec{r}^{'};\xi_n)=D_{lk}(\vec{r},\vec{r}^{'};\xi_n)-\bar{D}_{lk}(\vec{r},\vec{r}^{'};\xi_n) \qquad (2.3)$$

$$D^{'H}_{lk}(\vec{r},\vec{r}^{'};\xi_n)=rot_{l\mu}rot^{'}_{k\nu}D^{'}_{\mu\nu}(\vec{r},\vec{r}^{'};\xi_n) \qquad (2.4)$$

where

$$rot_{l\mu}\equiv\varepsilon_{lp\mu}\frac{\partial}{\partial x_p},rot^{'}_{k\nu}\equiv\varepsilon_{ks\nu}\frac{\partial}{\partial x^{'}_s} \qquad (2.5)$$

The force per unit area is given by the following formula

$$F(z_0)=\frac{kT}{4\pi}\sum_{n=0}^{\infty}{}'\{\frac{\xi_n^2}{c^2}\varepsilon\ (z_0)[D^{'}_{33}(.)-D^{'}_{11}(.)-D^{'}_{22}(.)]+D^{'H}_{11}(.)+D^{'H(.)}_{22}-D^{'H}_{33}(.)\} \qquad (2.6)$$

where $(.)\equiv(\vec{\rho},z_0;\vec{\rho},z_0)$ and the prime indicates that the n=0 term is to be multiplied by ½. Due to homogeneity in the x-y plane we introduce the following Fourier transform

$$D_{lk}(\vec{r},\vec{r}';\xi_n)=\int\frac{d^2\vec{q}}{4\pi^2}e^{i\vec{q}\cdot(\vec{\rho}-\vec{\rho}')}d_{lk}(\vec{q},\xi_n;z,z'),\ \ \vec{q}(q_x,q_y,0),\vec{\rho}(x,y,0) \qquad (2.7)$$

A further transformation is needed to replace the matrix $\overline{\overline{d}}$ with a new matrix $\overline{\overline{g}}$ depending only on $q=|\vec{q}|=(q_x^2+q_y^2)^{1/2}$ and of course of z,z',ξ_n:

$$d_{sj}=(S^T)_{ss'}g_{s'j'}(S)_{j'j} \qquad (2.8)$$

where

$$\overline{\overline{S}}(\vec{q})=\frac{1}{q}\begin{pmatrix} q_x & q_y & 0 \\ -q_y & q_x & 0 \\ 0 & 0 & q \end{pmatrix},\overline{\overline{S}}^T(\vec{q})=\frac{1}{q}\begin{pmatrix} q_x & -q_y & 0 \\ q_y & q_x & 0 \\ 0 & 0 & q \end{pmatrix} \qquad (2.9)$$

Then the Green's dyadic problem (2.1) reduces to

$$\overline{\overline{B}}\,\overline{\overline{g}}=4\pi\delta(z-z')\overline{\overline{I}} \qquad (2.10)$$

where $\left(\overline{\overline{I}}\right)_{lk}=\delta_{lk}$ is the unit 3x3 matrix and

$$\overline{\overline{B}}=\begin{pmatrix} -\varepsilon(z)\frac{\xi_n^2}{c^2}+\frac{\partial^2}{\partial z^2} & 0 & -iq\frac{\partial}{\partial z} \\ 0 & -\varepsilon(z)\frac{\xi_n^2}{c^2}-q^2+\frac{\partial^2}{\partial z^2} & 0 \\ -iq\frac{\partial}{\partial z} & 0 & -\varepsilon(z)\frac{\xi_n^2}{c^2}-q^2 \end{pmatrix} \qquad (2.11)$$

so that $\overline{\overline{g}}$ have only five non-zero elements

$$\overline{\overline{g}}=\begin{pmatrix} g_{11} & 0 & g_{13} \\ 0 & g_{22} & 0 \\ g_{31} & 0 & g_{33} \end{pmatrix} \qquad (2.12)$$

The equation for $g_{22}(z,z')$

$$\left[\frac{d^2}{dz^2}-q^2-\frac{\xi_n^2}{c^2}\varepsilon(z)\right]g_{22}(z,z')=4\pi\delta(z-z') \qquad (2.13)$$

corresponds to transverse electric (TE) waves (or s-polarization).At every point $z\neq z'$ we require continuity of the function and its derivative

$$g_{22}(z-0,z^{'}) = g_{22}(z+0,z^{'}), \left(\frac{\partial g}{\partial z}\right)_{z-0} = \left(\frac{\partial g}{\partial z}\right)_{z+0} \qquad (2.14)$$

For transverse-magnetic (TM) waves we derive the following equation for $g_{11}(z,z^{'})$:

$$\frac{d}{dz}\left(a(z)\frac{dg_{11}}{dz}\right) - \varepsilon(z)g_{11}(z,z^{'}) = 4\pi\frac{c^2}{\xi_n^2}\delta(z-z^{'}) \qquad (2.15)$$

where

$$a(z) = \frac{\varepsilon(z)}{w^2(z)}, w^2(z) = q^2 + \frac{\xi_n^2}{c^2}\varepsilon(z) \qquad (2.16)$$

and the boundary conditions to this equation are

$$g_{11}(z-0,z^{'}) = g_{11}(z+0,z^{'}), \left(a(z)\frac{dg_{11}}{dz}\right)_{z-0} = \left(a(z)\frac{dg_{11}}{dz}\right)_{z+0} \qquad (2.17)$$

For the other three functions corresponding to TM-waves we derive

$$g_{31}(z,z^{'}) = -\frac{iq}{w^2(z)}\frac{\partial g_{11}(z,z^{'})}{\partial z} \qquad (2.18)$$

$$g_{33}(z,z^{'}) = -\frac{iq}{w^2(z)}\frac{\partial g_{13}(z,z^{'})}{\partial z} - \frac{4\pi}{w^2(z)}\delta(z-z^{'}) \qquad (2.19)$$

$$\frac{d}{dz}\left(a(z)\frac{dg_{13}}{dz}\right) - \varepsilon(z)g_{13}(z,z^{'}) = -\frac{4\pi iqc^2}{\xi_n^2}\frac{d}{dz}\left(\frac{\delta(z-z^{'})}{w^2(z)}\right) \qquad (2.20)$$

3. Specific examples. VdW interactions between macroscopic bodies having inhomogeneous dielectric permittivities

Let us consider a physical system having the following distribution of the dielectric permittivity

$$\varepsilon(i\xi,z) = \begin{matrix} \varepsilon + \Delta\varepsilon\cos(2q_0 z), -\infty < z < -L/2 \\ \varepsilon_3, |z| < L/2 \\ \varepsilon + \Delta\varepsilon\cos(2q_0 z), L/2 < z < \infty \end{matrix} \qquad (3.1)$$

where we have denoted ξ_n with the symbol ξ.This is one-dimensional crystal in which an inclusion $|z| < L/2$ with permittivity $\varepsilon_3 = \varepsilon_3(i\xi)$is embedded. If the modulation function $\Delta\varepsilon = \Delta\varepsilon(i\xi) = 0$ we have the standard problem of interaction of two identical macroscopic bodies having permittivity $\varepsilon = \varepsilon(i\xi)$and separated by a plane parallel slab of thickness L.We will develop a perturbation theory for small modulation taking into account only linear with respect to $\Delta\varepsilon$ terms. Solving the equations (2.13) and (2.15) without the source $\sim\delta(z-z')$ terms we derive two dispersion relations $q=q(\xi)$ for existence of TE, respectively TM modes in the following form

$$\Delta_{TE}=1-e^{-2w_3L}\left(\frac{w_3-\tilde{w}}{w_3+\tilde{w}}\right)^2 \quad (3.2)$$

where

$$\tilde{w}=w\left[1+\frac{\Delta\varepsilon\xi^2}{2c^2(w^2+q_0^2)}(\cos(q_0L)-\frac{q_0}{w}\sin(q_0L))\right] \quad (3.3)$$

$$w=\left[q^2+\frac{\xi^2}{c^2}\varepsilon\right]^{1/2},w_3=\left[q^2+\frac{\xi^2}{c^2}\varepsilon_3\right]^{1/2} \quad (3.4)$$

$$\Delta_{TM}=1-e^{-2w_3L}\left(\frac{w\varepsilon_3-w_3\varepsilon p}{w\varepsilon_3+w_3\varepsilon p}\right)^2,p=1+2\alpha(\frac{q_0}{w}\sin(q_0L)-\cos(q_0L)) \quad (3.5)$$

$$\alpha=-\Delta\varepsilon(q_0^2+w^2)^{-1}\left[\frac{\xi^2}{4c^2}+\frac{q^2}{2\varepsilon}\right] \quad (3.6)$$

Formula (2.6) can also be written as $F(L)=-\partial G/\partial L$, where $|z_0|<L/2$ and the free energy of interaction per unit area is

$$G(L,T)=\frac{kT}{2\pi}\sum_{n=0}^{\infty}{}'\int_0^{\infty}q\ln\left[\Delta_{TE}\Delta_{TM}\right]dq \quad (3.7)$$

The result in equation (3.7) follows from the procedure given in Section 2 and is in accordance with calculations for homogeneous laminated media given earlier [7]. By making the substitution 2qL=x and also replacing the summation on n with with ξ-integration which is valid at low temperatures this becomes

$$E(L)=\frac{\hbar}{16\pi^2L^2}\int_0^{\infty}d\xi\int_0^{\infty}x\ln[\Delta_{TM}\Delta_{TE}]dx \quad (3.8)$$

We will analyze this result only in the nonretarded VdW limit (i.e., assuming an infinite speed of light) thus limiting application of our results to L no more than about $100\dot{A}$. In this limit $\Delta_{TE}\to 1$ and the contribution from the TM mode can be written as

$$\ln\Delta_{TM}(\xi,x;q_0L)\approx -e^{-x}(\frac{\varepsilon_3(i\xi)-\varepsilon(i\xi)}{\varepsilon_3(i\xi)+\varepsilon(i\xi)})^2+$$

$$\frac{4e^{-x}x^2(\cos(q_0L)-\frac{2q_0L}{x}\sin(q_0L))}{(4q_0^2L^2+x^2)(\varepsilon_3(i\xi)+\varepsilon(i\xi))^3}\varepsilon_3(i\xi)\Delta\varepsilon(i\xi)(\varepsilon_3(i\xi)-\varepsilon(i\xi)) \quad (3.9)$$

The first term (negative energy) corresponds to attraction forces whereas the second term has an oscillatory behavior as a function of $q_0 L$. Recently such oscillations with the thickness of the film has also been predicted in cholesteric crystalline films [8]. Integration with respect of x and ξ in (3.8) is elementary if we use some empirical characterization of all permittivities entering in (3.9) like [9] (here a=const., the dimension of b is rad/sec)

$$\varepsilon(i\xi) = \frac{1 + ae^{-b\xi}}{1 - ae^{-b\xi}} \qquad (3.10)$$

We derive

$$E(L) = \frac{\hbar}{16\pi^2 L^2}$$

$$\left[-\left(\frac{a_3^2}{2b_3} + \frac{a^2}{2b} - \frac{2aa_3}{b + b_3}\right) + \frac{\phi(q_0 L)}{4}\left(\frac{a_3}{b_3} - \frac{a}{b} + \frac{a_3^2}{b_3} + \frac{2a_3 \Delta a}{b_3 + \Delta b} - \frac{2a\Delta a}{b + \Delta b} - \frac{2a_3 a}{b_3 + b}\right) \right] \qquad (3.11)$$

where

$$\phi(q_0 L) = \begin{pmatrix} 1 - \frac{5}{2}(q_0 L)^2, q_0 L \ll 1 \\ -\frac{\sin(q_0 L)}{q_0 L} + \frac{3\cos(q_0 L)}{2(q_0 L)^2}, q_0 L \gg 1 \end{pmatrix} \qquad (3.12)$$

References

1. A. Ashkin, Phys. Rev. Lett. 25, 1321 (1970).
2. K. Visscher, G.J. Brakenhoff, Optik (Stuttgart) 89, 174 (1992).
3. P.C. Chaumet, M. Nieto-Vesperinas, Phys. Rev. B61, 14119 (2000).
4. K.A. Milton, J. Phys. A37, R209-R277 (2004).
5. I.E. Dzyaloshinskii, E. M. Lifshitz, L.P. Pitaevskii, Adv. Phys. 10, 165 (1961).
6. E. M. Lifshitz, L.P. Pitaevskii “Statistical Physics.part 2.” (New York, Pergamon, 1986).
7. N.G. van Kampen, B.R.A. Nijboer, K. Schram, Phys. Lett. 26A, 307 (1968).
8. I.E. Dzyaloshinskii, E. I. Kats, J. Phys.: Condens. Matter 16, 5659 (2004).
9. H. Krupp, Adv. Colloid Interface Sci. 1, 111 (1967).