CLASS OF INHOMOGENEOUSLY DRIVEN DYNAMICAL SYSTEMS: GENERAL THEORY, REGULAR AND CHAOTIC PROPERTIES

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Abstract:
A generalized model of an oscillator, subjected to the influence of an external wave is considered. It is shown that the systems of diverse physical background which this model encompasses by their nature should belong to the broader class of "kick-excited self-adaptive dynamical systems".

INTRODUCTION

The main goal of this report is to present a phenomenon of highly general nature manifested in various dynamical systems. We present the occurrence of peculiar "quantization" by the parameter of intensity of the excited oscillations (See the References below [1-9]). Quantization (the idea of quantums, photons, phonons, gravitons) is postulated in Quantum Mechanics, while Theory of Relativity does not derive quantization from geometric considerations. In the case of the established phenomenon the "quantized nature" of portioned energy transfer stems directly from the mechanisms of the process and has a precise mathematical description.

Here we also consider the generalized "oscillator-wave" model [10] and show that, in this case, the inhomogeneous external influence is realized naturally and does not require any specific conditions.

1. **A NUMERIC DEMONSTRATION** OF THE EXCITATION OF "QUANTIZED" PENDULUM OSCILLATIONS

The inhomogeneously AC driven, damped pendulum system can be described by the following equation:

$$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \sin x = \varepsilon(x)F_o \sin \nu t\,, \qquad (1)$$

where the function $\varepsilon(x)$ can be analytically expressed in various ways, such as:

$$\varepsilon(x)=\begin{cases}1 & for \quad |x|\le d' \\ 0 & for \quad |x|>d'\end{cases} \qquad (2)$$

or $\varepsilon(x)=e^{-\frac{x^2}{2d'^2}}$, $\varepsilon(x)=\begin{cases}\frac{1}{2}\left[1+\cos(\frac{\pi}{d'}x)\right] & for \quad |x|\le d' \\ 0 & for \quad |x|>d'\end{cases}$ and so on; $d'<<1$ is a parameter limiting the external action on a small part of the trajectory of motion in the system (in the particular case(2) the parameter d' thereby determines a symmetric zone of action in the area of the lower equilibrium position).

Results from the computer experiment are presented in [1,2].

2. ANALYTICAL PROOF OF EXISTENCE OF KICK-PENDULUM "QUANTIZED" OSCILLATIONS

The analytic approach for the cases of small and large amplitudes of pendulum oscillation is given in [1,2,3,4,5,6,7,8,9].

The spectrum of the symmetrical solution amplitudes can be expressed as [2,7]: $2vK(k_o)=2\pi\left(l+\frac{1}{2}\right)$, l= 0, 1, 2, 3…, which yields the spectrum of amplitudes k_o, $K(k_o)=\left(l+\frac{1}{2}\right)\frac{\pi}{v}$, l= 0, 1, 2, 3… and an odd ratio of frequency division N = 2l + 1, l= 0, 1, 2, 3…, $K(k_o)=\left(l+\frac{1}{2}\right)\frac{\pi}{v}$ is complete normal elliptic integral of first kind.

A phenomenon of rotation excitation with strictly determined discrete set of possible rates is presented. The model considered is that of a pendulum rotator under inhomogeneous action [1,2].

General conditions for pendulum oscillation excitation under the action of external nonlinear force has been derived [1,2,9].An analytical approach for large amplitudes of pendulum oscillation (strong nonlinearity) has been also demonstrated [1,2,5,6,7].

3. MODEL OF THE INTERACTION OF AN OSCILLATOR WITH AN FALLING WAVE

Analytical approaches applicable for small and large amplitudes (for weak and strong nonlinearity) of the oscillations in a nonlinear dynamic system subjected to the influence of a wave has been developed [1,10].

The performed analysis shows that the continuous wave having a frequency much larger than the frequency of a given oscillator can excite in it oscillations with a frequency close to its natural frequency and an amplitude belonging to a discrete set of possible stable amplitudes.

4. “QUANTIZED” CYCLOTRON MOTION

An electric charge, moving on a circular orbit in a homogeneous permanent magnetic field has been considered. When the charge was irradiated by a flat electromagnetic wave having a length commensurable with the orbit’s radius, an effect of discretization (“quantization”) of the possible stable orbital radii (or motion velocities) was observed (See [1,10]).

5. THE WAVE NATURE AND DYNAMICAL QUANTIZATION OF THE SOLAR SYSTEM

The Solar system planets mean distances are presented (See [1,10]). For comparison reasons, the direct astronomic measurements data is given parallel to the result, computed by the classical Titius-Bode law (11) and an Equation according to the “oscillator-wave” model, described above. A good correspondence is observed between the computed and astronomically measured radii. Especially significant is the correspondence between the computed and measured radii of Neptune and Pluto. The Titius-Bode law determines the mean distances of those two planets with an error of 23% and 49%, respectively.

The computed data of the mean satellite distances from Saturn, Uranus and Jupiter, as well as the mean ring system distances from Saturn, are also presented (See [1,10]). The calculations are made on the basis of the “oscillator-wave” model. Again, a good correspondence is seen between the calculated and measured mean distances [1,10].

Assumption that the Solar system is e a wave dynamic system and hence, the micro-mega-analogy (MM-analogy) is valid [11], is the essence and grounds for the presented consideration.]

6. GENERAL CONDITIONS FOR TRANSITION TO IRREGULAR BEHAVIOUR IN AN OSCILLATOR UNDER WAVE ACTION

General conditions for transition to irregular and chaotic behaviour in an oscillator under wave action have been derived using the notion about the Melnikov distance [1,10].

7. GENERAL CHARACTERISTIC FEATURES OF THE CLASS OF KICK-EXCITED SELF-ADAPTIVE DYNAMICAL SYSTEMS. CONCLUSIONS

The main characteristics and regularities, characterizing the considered class of kick-excited self-adaptive dynamical systems are as follows:

1. The excitation of oscillations with a quasi-natural system frequency and numerous discrete stationary amplitudes, depending only on the initial conditions.

2. Adaptive self-control of the energy contribution in the oscillating process.

Regardless of its simplicity, the "oscillator-wave" model obviously reflects a number of processes in the micro- and macro-world.

On the basis of the presented oscillator-wave model it is also possible to create heuristic models of the interaction of electromagnetic waves with plasma particles in the Earth's ionosphere and magnetosphere, heuristic models of the generation of powerful low-frequency waves in the space around the Earth when a cosmic electromagnetic background is present etc. High-efficient sub-millimeter emitter, built on this basis, could be suitable for radio-physical heating of plasma, e.g. in the experiments aimed the achievement of controllable thermonuclear reaction [1].

The method developed of entering energy in oscillation processes and the excitation of "quantized" oscillations in dynamic macro-systems finds and will find in the future applications which could be grouped in the following way:

1. Transformation of signals and oscillations of different nature by frequency with a high efficiency at single division of the frequency by ratio of tens, hundreds and thousands.

2. Energy transformation of one kind into another, for example of electric into mechanical and vice-versa.

3. Stabilization of different parameters with their change in a wide range (e.g. 50-100-300%).

4. The development of new base elements for specialized calculating devices possessing a large number of stable discrete states.

5. Intensification of different processes through a special organization of interaction of different oscillation or development of different wave technologies.

6. The modelling of micro- and macro- processes with the methods of classic oscillation theory.

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