Gravity-Darkening in Semi-Detached Binary Systems TW And, TW Cas, AI Dra and UX Her

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INTRODUCTION

In this paper we are dealing with the estimation of the gravity-darkening exponent (GDE, $\boldsymbol{\beta}$), which describes the dependence of the emergent flux of total radiation on the local gravity acceleration of a rotationally or tidally distorted star in hydrostatic equilibrium. Theoretical predictions for stars with purely radiative energy transfer give the value of $\boldsymbol{\beta} = 0.25$ [1], while for stars with convective envelopes $\boldsymbol{\beta} = 0.08$ [2]. Recently however, it was suggested that both mechanisms could act simultaneously [3], and thus any value between these two extremes is expected.

Several attempts have been made to estimate the value of $\boldsymbol{\beta}$ from observational data for various kinds of binary systems (see e.g. [4] and references therein).

In a semi-detached binary system the secondary component fills its Roche lobe, while the primary is well deep inside its own Roche lobe. In the analysis of such a system it is reasonable to fix the GDE value of the primary star to its theoretical value. The appropriate GDE value for the Roche lobe-filling component could be then empirically estimated from observational data by light-curve analysis.

RESULTS

Here we have performed such an analysis on four semi-detached binary systems and estimated the GDE values for their Roche lobe filling components. More details on the applied model and method of light-curve analysis can be found in [4].

The binary systems in question are: *TW And* (F0V + K1-3III-IV; P ~ $4^d.12$) [5] with the mass ratio q = 0.193 estimated from radial-velocity measurements by [6]; *TW Cas* (B9 + G; P ~ $1^d.428$) [7], [8] with q = 0.38 [9]; *AI Dra* (B9.6 V+ G2?; P ~ $1^d.19$; q = 0.43) [10], and *UX Her* (A2.6 + K; P ~ $1^d.549$; q = 0.248) [11].

Tables 1 and 2 comprise the results of our light-curve analyses, and Fig.1 and 2 give the graphic presentation of the obtained results. The subscripts (h,c) in the tables denote the hotter and cooler component of the system. The *Note* given below Table 1 is also valid for Table 2. For *TW And*, *TW Cas*, *AI Dra* and *UX Her* we have obtained β_c ~ 0.06, β_c ~ 0.13, β_c ~ 0.12, and β_c ~ 0.06, respectively. These values basically confirm theoretical predictions for stars with convective envelopes.

System Quantity	TW And B-filter	TW And V-filter	TW Cas B-filter	TW Cas V-filter
n	244	247	647	649
Σ (O-C)^2	0.0265	0.0244	0.0597	0.0485
σ	0.0104	0.0100	0.0096	0.0087
$q = m_c / m_h$	0.1928		0.38	
T_h	7200		12000	
β_h	0.25		0.25	
A_h	1.0		1.0	
A_c	0.5		0.5	
$f_h = f_c$	1.0		1.0	
T_c	4395±13	4466±14	5857±45	5977±29
F_h	0.322±0.001	0.321±0.001	0.583±0.003	0.584±0.003
F_c	0.999±0.001	1.000±0.001	0.994±0.001	0.994±0.001
i [^{o}]	87.17±0.03	87.12±0.04	75.66±0.03	75.64±0.03
β_c	0.05±0.01	0.06±0.01	0.12±0.02	0.13±0.02
$a^{h,c}{}_1$	+0.3243,+0.5289	+0.3772,+0.6891	+0.5853,+0.3929	+0.6396,+0.4350
$a^{h,c}{}_2$	+1.0356,-0.6472	+0.9587,-0.8787	+0.4825,+0.1973	+0.0445,+0.4293
$a^{h,c}{}_3$	-0.7918,+1.3420	-0.9546,+1.7082	-0.5779,+0.5639	-1.0004,+0.0051
$a^{h,c}{}_4$	+0.2444,-0.2800	+0.3466,-0.6172	+0.2072,-0.2901	+0.0317,-0.0885
Ω_h	6.540	6.551	4.291	4.289
Ω_c	2.216	2.215	2.645	2.645
R_h [D=1]	0.157	0.157	0.255	0.255
R_c [D=1]	0.230	0.230	0.277	0.277
$L_h / (L_h+L_c)$	0.871	0.789	0.922	0.872
$\square_h$ [M_s]	1.68±0.07		3.97±0.05	
$\square_c$ [M_s]	0.32±0.02		1.51±0.04	

□ $_h$ [R_s]	2.15±0.05	2.42±0.03
□ $_c$ [R_s]	3.38±0.09	2.79±0.03
log g_h	4.00±0.02	4.27±0.02
log g_c	2.89±0.02	3.73±0.02
M^h_{bol}	2.17±0.02	-0.31±0.02
M^c_{bol}	3.30±0.05	2.46±0.05
a_{orb} [R_s]	13.64±0.22	9.40±0.20

Table 1. Results of the analysis of *TW And* & *TW Cas B* and V light curves obtained by solving the inverse problem for the Roche model. Gravity darkening exponent of the cooler secondary component (***β_c***) was a free parameter.

Note: Black-body approximation of stellar atmosphere, n – number of observations, Σ $(O\text{-}C)^2$ – final sum of squares of residuals between observed and synthetic light curves, σ – standard deviation of the observations, $q = m_c / m_h$ – mass ratio of the components, $T_{h,c}$ – temperature of the hotter primary and cooler secondary, $\beta_{h,c}$, $A_{h,c}$, $f_{h,c}$ – gravity-darkening exponents, albedos and non-synchronous rotation coefficients of the components respectively, $F_{h,c}$ – filling factors for the critical Roche lobe of the hotter primary and cooler secondary, i [^{o}] – orbit inclination (in arc degrees), $a^{h,c}_1$, $a^{h,c}_2$, $a^{h,c}_3$, $a^{h,c}_4$ – non-linear limb-darkening of the components (Claret's formula [12]), $\Omega_{h,c}$ – dimensionless surface potentials of the components, $R_{h,c}$ – polar radii of the components in units of distance between the component centers, $L_h / (L_h+L_c)$ – luminosity of the more massive hotter star, □ $_{h,c}$ [M_s], □ $_{h,c}$ [R_s] – stellar masses and mean radii of stars in solar units, log $g_{h,c}$ – logarithm (base 10) of the system components effective gravity, $M^{h,c}_{bol}$ – absolute bolometric magnitudes of the components, and a_{orb} [R_s] – orbital semi-major axis in units of solar radius.

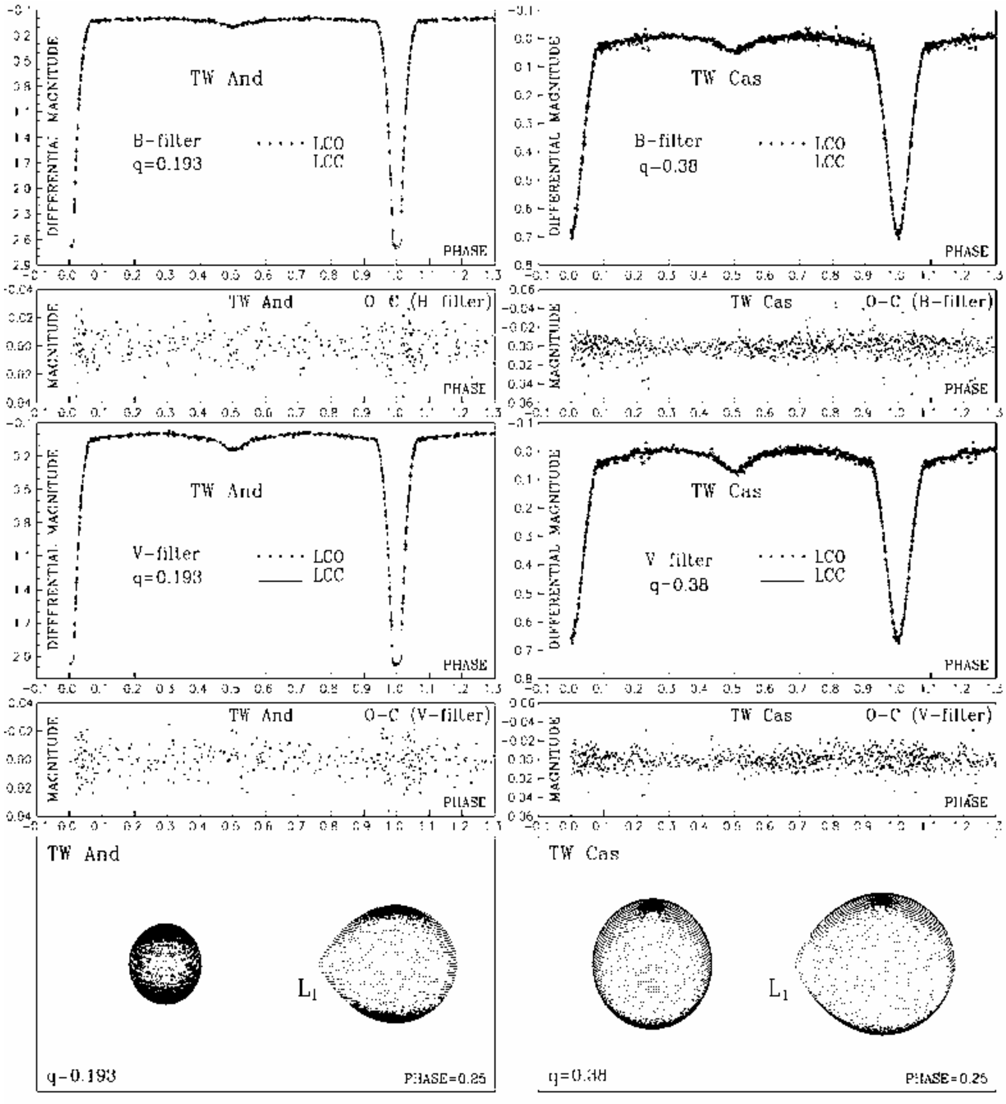

Figure 1. Observed (LCO) and final synthetic (LCC) light curves of *TW And* & *TW Cas*, with final O-C residuals obtained by analyzing their observations, and the views of the systems at the orbital phase 0.25 obtained with parameters estimated in the light-curve analysis.

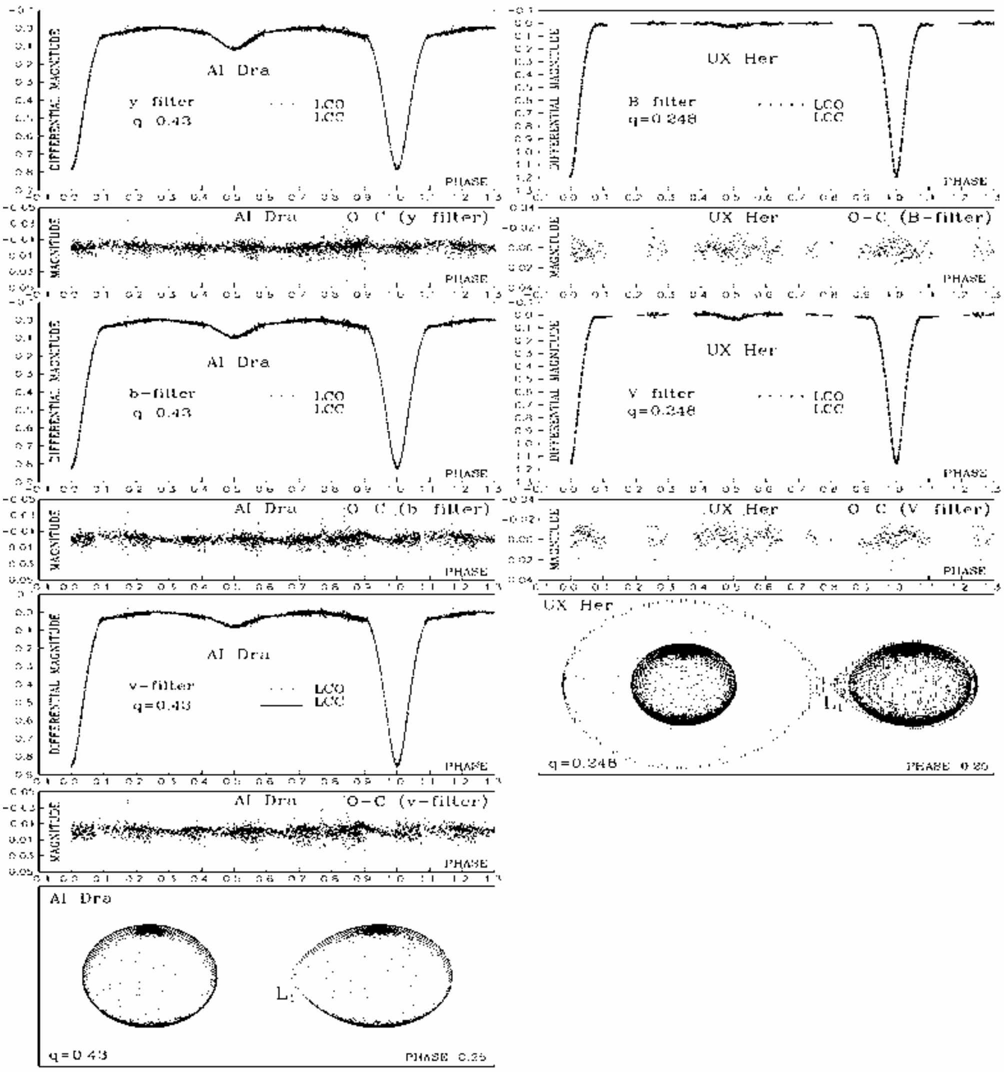

Figure 2. Observed (LCO) and final synthetic (LCC) light curves of *AI Dra* & *UX Her*, with final O-C residuals obtained by analyzing their observations, and the views of the systems at the orbital phase 0.25 obtained with parameters estimated in the light-curve analysis.

System Quantity	AI Dra y-filter	AI Dra b-filter	AI Dra v-filter	UX Her B,V-filters
n	2635	2635	2635	696
Σ (O-C)^2	0.0862	0.0927	0.1213	0.0344
σ	0.0057	0.0059	0.0068	0.0071
$q = m_c / m_h$	0.43			0.248±0.005
T_h	9800			9000
β_h	0.25			0.25
A_h	1.0			1.0
A_c	0.5			0.5
$f_h = f_c$	1.0			1.0
T_c	5607±15	5550±16	5445±20	4055±22
F_h	0.670±0.001	0.669±0.001	0.670±0.001	0.483±0.001
F_c	0.990±0.001	0.992±0.001	0.993±0.001	0.931±0.001
i [^{o}]	77.40±0.08	77.37±0.08	77.42±0.08	82.12±0.01
β_c	0.120±0.007	0.123±0.007	0.118±0.009	0.06±0.02
$a^{h,c}{}_1$	+0.5543,+0.5400	+0.4513,+0.4155	+0.4390,+0.5413	+0.4027,+0.5209 [B]
				+0.5877,+0.7136 [V]
$a^{h,c}{}_2$	+0.3222,-0.1002	+0.8004,+0.0291	+0.8980,-0.6903	+0.7907,+0.3053 [B]
				-0.6993,-0.9308 [V]
$a^{h,c}{}_3$	-0.2696,+0.8016	-0.7284,+0.8797	-0.8103,+1.7817	-0.5422,-0.1246 [B]
				+1.1472,+1.5821 [V]
$a^{h,c}{}_4$	+0.0717,-0.4229	+0.2283,-0.4501	+0.2534,-0.7090	+0.1238,-0.0076 [B]
				-0.0969,-0.4541 [V]
Ω_h	3.911	3.915	3.910	4.641
Ω_c	2.755	2.751	2.750	2.427
R_h [D=1]	0.286	0.286	0.286	0.227
R_c [D=1]	0.285	0.286	0.286	0.231
$L_h / (L_h+L_c)$	0.867	0.902	0.933	0.981 [B]; 0.973 [V]
$\square_h$ [M_s]	2.79±0.02			2.28±0.09
$\square_c$ [M_s]	1.20±0.02			0.56±0.11
$\square_h$ [R_s]	2.19±0.02			1.82±0.02
$\square_c$ [R_s]	2.30±0.02			1.94±0.02
log g_h	4.20±0.02			4.27±0.02
log g_c	3.79±0.02			3.62±0.02
$M^h{}_{bol}$	0.79±0.02			1.56±0.02
$M^c{}_{bol}$	3.17±0.06			4.89±0.06
a_{orb} [R_s]	7.521±0.006			7.97±0.01

Table 2. Results from the analysis of *AI Dra* (Strömgren *ybv*) light curves, and *UX Her* (BV) light curves obtained by solving the inverse problem for the

Roche model. Gravity darkening exponent of the cooler secondary component (***β_c***) was a free parameter.

Note: The labels are the same as in Table 1.

DISCUSSION

The light-curve analysis of the four semi-detached binary systems (*TW And*, *TW Cas*, *AI Dra*, *UX Her*) presented here, made within the Roche model and gave us the ***real possibility*** to estimate the GDE values for their secondary components ***without any additional approximations***. During the same analyzing procedure, we can estimate the systems' parameters, too, (as they are given at Tables 1 & 2). The estimated values of GDE basically confirm the theoretical predictions for stars with convective envelopes.

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