BIFURCATION SOLUTIONS OF VORTICAL TRANSPORT EQUATION PARAMETERS, AS A MECHANISM OF DETECTING AN INSTABILITY IN THE ACCRETION DISCS

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Abstract

The vortices play a very important role for the existence of accretion discs. Usually they arise under the influence of different kind of instabilities. One basic equation that describes these processes is the reaction-diffusion equation and its prototype in our case – vortical transport equation. Here using the bifurcation analyses of the presented equation we receive some bifurcation solutions, which are the mechanism of the vortical formations, locally in the accretion discs.

Introduction

The vortical equation received from consecutive transformations of the basic hydro-dynamical equations is our analog of the reaction-diffusion type equation for the accretion discs. The equation, which we received [1], has some difference in the expression from the well known vortical equation [4].

It is follows from the fact that we represent the reaction-diffusion equation with quantities, describing the accretion disc. We consider the terms and processing in thin non-axisymetrical accretion discs.

In this paper we will turn our view on the ruling parameter, which variation and extremum are the reason for the arising of instability. We have to determine here this parameter for the vortical transport equation and find the bifurcation solution for the different kind of bifurcation.

The bifurcation changes the qualitative behavior of the system, when the parameter of the system reached its critical value [5], which is known as a bifurcation point. Passing trough this point, the system turns from stable state to unstable state or to another stable situation.

For most cases, the analytical and numerical results show [3] that the locations of fixed points are determined by the parameters and these fixed points are either structurally stable or unstable.

Analyses and results

I. Bifurcations and amplitude equations.

It is used in the bifurcation analyses a specifically quantities and non-linear equations. We mentioned above for the critical value of the control parameter of the system. We denote it here with λ_c, which is in fact a bifurcation point. When the given system passed trough this bifurcation point, it may lose its stability and there are conditions to forming structures in these places. The obtaining results have to be of global character, but they have an effect of local behavior.

When we talk about bifurcation, we have to specify that there are exist a different kind of its evince. The essence of different kind of bifurcation is defined from relevant amplitude equation. We present in this section amplitude equations of basic type of bifurcations.

1. Transcritical bifurcation:

$\frac{dw}{dt} = (\lambda - \lambda_c)Q_1 w - Q_2 w^2$ There are two fixed points and the solution is:

$$w_{S0} = 0, w_{S1} = (\lambda - \lambda_c)\frac{Q_1}{Q_2}$$

2. Pitchfork bifurcation:

$\frac{dw}{dt} = (\lambda - \lambda_c)Q_1 w - Q_3 w^2$ The fixed points in this case are three:

$$w_{S0} = 0, w_{S1} = \pm[(\lambda - \lambda_c)\frac{Q_1}{Q_3}]^{1/2}$$

3. Hoph bifurcation:

$$\frac{dw}{dt} = (\lambda - \lambda_c)Q_1 w - Q_3 |w|^2 w$$

Here the coefficients Q_1 and Q_3 are complex quantities and they are expressed as:

$Q_1 = Q'_1 + iQ''_1$ and $Q_3 = Q'_3 + iQ''_3$

The Hoph bifurcation has a periodically character. We have to introduce here for the amplitude w polar coordinates, so: $w = qe^{i\phi}$

Because of this we receive the solutions for q and they are in the form:

$$q_{S0} = 0, q_{S1} = \pm\left[\frac{\left(\lambda - \lambda_c Q'_1\right)}{Q'_3}\right]^{1/2}$$

The basic denotations in this section:

λ - the control parameter;

λ_c - the critical value of the control parameter;

Q_1, Q_2, Q_3 - the numerical coefficients, determined entirely by the structured of the initial equations and solutions;

II. Basic equations, vortical transport equation.

In our examination for the accretion discs we have to determine the coefficients Q_1, Q_2, Q_3 from the solution of the next system of equations. These are the basic equations being in the studying of the accretion discs dynamics.

1. The first equation is the well known from the hydrodynamics continuity equation:

$$(5) \quad \frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0$$

2. The second two equations present the development of the Navier-Stokes equation in cylindrical coordinates in the terms of r, φ:

$$(6)\quad \frac{\partial v_\varphi}{\partial t} + v_r \frac{\partial v_\varphi}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi} - \frac{v_r\ v_\varphi}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \varphi} + \frac{1}{\rho}F_\varphi + \nu\left(\frac{\partial^2 v_\varphi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_\varphi}{\partial \varphi^2} + \frac{1}{r}\frac{\partial v_\varphi}{\partial r} - \frac{2}{r^2}\frac{\partial v_r}{\partial \varphi} - \frac{v_\varphi}{r^2}\right)$$

Where $F_\varphi = \frac{1}{2}(\Omega \times r)^2$

$$(7)\quad \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_r}{\partial \varphi} - \frac{{v_\varphi}^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{1}{\rho}F_r + \nu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \varphi^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{2}{r^2}\frac{\partial v_\varphi}{\partial \varphi} - \frac{v_r}{r^2}\right)$$

Where $F_r = \frac{1}{2}(\Omega \times r)^2 + G\frac{m_1 m_2}{r_1 + r_2}$

3. The energy conservation equation here is in the form:

$$(8)\quad \frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\varepsilon + \rho g z\right) + div\left[\left(\frac{1}{2}\rho v^2 + \rho\varepsilon + P\right)v\right] = 0$$

where $\frac{1}{2}\rho v^2$ - the kinetic energy per unit volume,

$\rho\varepsilon$ - is the internal or thermal energy per unit volume.

$\rho g z$ - is the potential energy / z is the unit vector over z-direction /

4. We represent here our view of vortical transport equation, which is in fact a vortical evolution law. We obtain this equation after taking the curl of Navier-Stokes equation and some transformations of the reaction-diffusion equation.

(9) $$\frac{\partial\Psi}{\partial t} = (\nabla . v_r)\Psi_r - (\nabla . \Psi_\varphi)v_\varphi + \frac{D_r}{D_\varphi}\nabla^2\Psi$$

5. Because of the important role of angular momentum in the accretion discs, the next equation presents it the form:

(10) $$\frac{\partial}{\partial t}(\rho r v_\varphi) + \nabla . r\rho v_\varphi + \nabla . rP - \nabla . \left[\frac{1}{3} r v (\nabla . v) + v r^2 \nabla \frac{v_\varphi}{r}\right] = 0$$

Where the denotations for all equations are:

ρ - is the mass density of the flow;

v - is the velocity of the flow / over r and φ direction /;

P - is the pressure;

Ψ - is the vorticity / over r and φ direction /;

r - is the radius of the disc;

D - is the diffusion coefficient (or matrix of the transport coefficient);

III. A graphical view of the bifurcation solutions.

After receiving the solutions for Ψ, D, v, r we use them to express the bifurcation equations in the term of the above equations. We show here the graphical view of the new form of these equations:

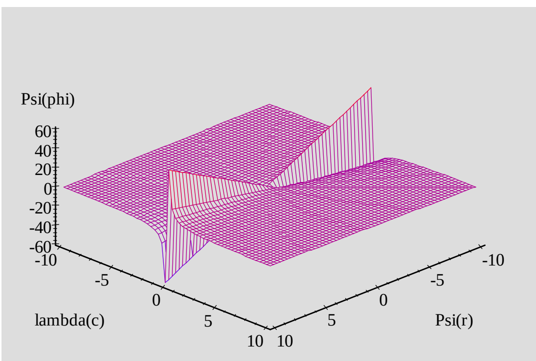

Figure 1. Graphical solutions of the transcritical bifurcation.

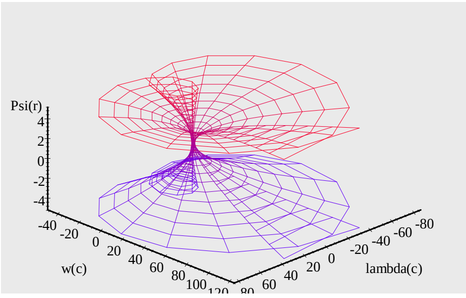

Figure 2. A graphical view of th pitchfork bifurcation solutions

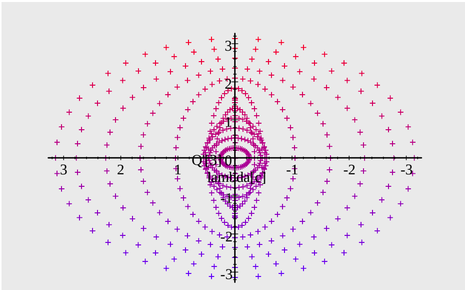

Figure 3. Graphical solutions of the Hoph bifurcation.

Conclusion

We presented here the behavior of bifurcation control parameter in the terms and solutions of the basic hydrodynamical equations and vortical transport equation, ruling the accretion disc flow. We showed the graphical view of this parameter for the three types of bifurcation. It is important to note that in the places where the bifurcation acts, there are conditions for arising of instability. This instability may give rise to structure formations, locally in accretion disc. Such structures, as vortical formations play an important role in the angular momentum transfer.

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