THREE DIMENSIONAL NUMERICAL MODEL OF ACCRETION FLOW IN CLOSE BINARY – PROBLEM DEFINITION

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Abstract

We present the basic equations and approximations in descriptions of accretion flow in close binary system. Then we discuss the most useful geometry for numerical simulations. We introduce to numerical method and program language we choose. Finally we discuss the reason for building such a model.

Basic Equations

The accretion flow is a gas flow, which moves in the gravitational and magnetic fields of both stars and trough the there radiation.

The basic equation described this motion are:

1. The equation of mass conservation:

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho V\right) = 0$$

2. The equations of motion (one for each space coordinate).

$$\rho \frac{\partial V}{\partial t} + \rho V.\nabla V = -\nabla p + F$$

3. The equation of energy conservation.

$$\frac{\partial}{\partial .t}\left[\varepsilon + \frac{V^2}{2}\right] = -V\nabla\left[\varepsilon + \frac{V^2}{2}\right] + div\left[\lambda.gradT + V.\sigma + W\right]$$

4. The equation of state

$p = p(\rho, T)$

5. The equation of radiation transfer:

$$W = \int_0^\infty d\nu \int_{4\pi} \Omega I_\nu d\Omega$$

We use the approximation of irradiative thermo-conduction. It means that we accept that the flow is optically thick. In this case (Chetverushkin 1985):

$W=\chi T^3.\mathrm{grad}T$

To complete the system we need to know the forces and the descriptions of the viscosity and temperature exchange.

As in our previous investigations, we will use functions for viscosity of this kind:

$\nu(\rho, T) = \nu_0..\rho^a.T^b$, $\chi(\rho, T) = \chi_0..\rho^c.T^d$, $\lambda(\rho, T) = \lambda_0..\rho^e.T^f$

and the equation of state for perfect gas.

We build our model for binary systems with weak magnetic fields and do not involve the electromagnetic forces in equations.

The geometry

In the case of close binary, the gas flow from primary star moves toward the secondary and rotate around it. In the same time, the binary ac whole rotates with its own angular velocity around the mass center of the system.

This gives us a reason to choose a non-inertional cylindrical reference frame with the center – secondary star, which rotates with velocity, equal to those of the binary system.

In this frame, the equations of motion of a fluid with small change of viscosity from point to point becomes as follows (Landau, Lifshic 1986):

$$\frac{\partial \rho}{\partial t} = -\rho . \left(\frac{\partial V_r}{\partial r} + \frac{\partial V_\varphi}{r \partial \varphi} + \frac{\partial V_z}{\partial z} + \frac{V_r}{r} \right)$$

$$\frac{\partial V_r}{\partial t} = -V_r \frac{\partial V_r}{\partial r} - \frac{V_\varphi}{r} \frac{\partial V_r}{\partial \varphi} - V_z \frac{\partial V_r}{\partial z} + \frac{{V_\varphi}^2}{r} - \frac{1}{\rho} \frac{\partial P}{\partial r} + F_r +$$
$$\nu \left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 V_r}{\partial \varphi^2} + \frac{\partial^2 V_r}{\partial z^2} \frac{1}{r} \frac{\partial V_r}{\partial r} - \frac{2}{r^2} \frac{\partial V_\varphi}{\partial \varphi} - \frac{V_r}{r^2} \right)$$

$$\frac{\partial V_\varphi}{\partial t} = -V_r \frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r} \frac{\partial V_\varphi}{\partial \varphi} - V_z \frac{\partial V_\varphi}{\partial z} + \frac{V_r V_\varphi}{r} - \frac{1}{\rho r} \frac{\partial P}{\partial \varphi} + F_\varphi +$$
$$\nu \left(\frac{\partial^2 V_\varphi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 V_\varphi}{\partial \varphi^2} + \frac{\partial^2 V \varphi}{\partial z^2} + \frac{1}{r} \frac{\partial V_\varphi}{\partial r} - \frac{2}{r^2} \frac{\partial V_r}{\partial \varphi} - \frac{V_\varphi}{r^2} \right)$$

$$\frac{\partial V_z}{\partial t} = -V_r \frac{\partial V_z}{\partial r} - \frac{V_\varphi}{r} \frac{\partial V_z}{\partial \varphi} - V_z \frac{\partial V_z}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial z} + F_z +$$
$$\nu \left(\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 V_z}{\partial \varphi^2} + \frac{1}{r} \frac{\partial V_z}{\partial r} - \frac{\partial^2 V_z}{\partial z^2} \right)$$

$F = F_{g1} + F_{g2} + F_{cr}$

Here each space component of F contains gravitational and magnetic forces from both stars and centrifugal force. Gravitational forces are:

$$F_{g1} = \frac{GM_1}{(r^2 + R_{12}^2 - 2rR_{12} \cos \varphi + z^2)^{1/2}} \qquad F_{g2} = \frac{GM_2}{(r^2 + z^2)^{1/2}}$$

And its r, φ and z components are:

$$F_{g1}(r,\varphi) = F_{g1}\cos\beta \qquad F_{g1}(z) = F_{g1}\sin\beta$$

$$F_{g1}(\varphi) = F_{g1}(r,\varphi)\sin(\varphi+\gamma) \qquad F_{g1}(r) = F_{g1}(r,\varphi)\cos(\varphi+\gamma)$$

$$F_{cr} = \Omega^2 R_{cr} \qquad \Omega = (\frac{G(M_1+M_2}{R_{12}^3})^{1/2}$$

$$R_2 = \frac{M_2}{M_1+M_2}R_{12} \qquad R_{cr} = (r^2 + R_2 - 2rR_2\cos\varphi)^{1/2}$$

F_{cr} has only radial and tangential components, which we can write using similar picture

$$F_{cr}(r) = F_{cr}(r,\varphi)\cos(\varphi+\beta) \qquad F_{cr}(\varphi) = F_{cr}(r,\varphi)\sin(\varphi+\beta)$$

Using equation for perfect gas

$$p = \frac{R}{\mu}\rho T \qquad \varepsilon = \int_0^T c_v dT \Rightarrow d\varepsilon = c_v dT$$

$$\frac{\partial T}{\partial t} = -V\nabla T - \frac{1}{c_v}\left\{\frac{d}{dt}\left[\frac{V^2}{2}\right] + div\left[\left(\lambda+\chi.T^3\right)gradT + V.\sigma\right]\right\} = -V\nabla T - \frac{V}{c_v}\frac{dV}{dt} +$$
$$\frac{1}{c_v}\left\{div\left[\left(\lambda+\chi.T^3\right)gradT + V.\sigma\right]\right\} = -V\nabla T - \frac{V}{c_v}\frac{dV}{dt} + \frac{1}{c_v}\left\{\chi.T^3 div.gradT + 3\chi.T^2.grad^2T + div(V.\sigma)\right\}$$

Finally in most cases we can accept that we can neglect all second derivatives, so we find:

$$\frac{\partial T}{\partial t}=-V_r\frac{\partial T}{\partial r}+\frac{V_\varphi}{r}\frac{\partial T}{\partial \varphi}+V_z\frac{\partial T}{\partial z}+\frac{1}{c_v}\left\{\ .V_r\left[\frac{1}{\rho}\frac{\partial P}{\partial r}+\frac{1}{\rho}F_r+\nu\left(-\frac{1}{r}\frac{\partial V_r}{\partial r}-\frac{2}{r^2}\frac{\partial V_\varphi}{\partial \varphi}-\frac{V_r}{r^2}\right)\right]\right.$$

$$+\frac{.V\varphi}{r}\left[\frac{1}{\rho r}\frac{\partial P}{\partial \varphi}+\frac{1}{\rho}F_\varphi+\nu\left(+\frac{1}{r}\frac{\partial V_\varphi}{\partial r}-\frac{2}{r^2}\frac{\partial V_r}{\partial \varphi}-\frac{V_\varphi}{r^2}\right)\right]+V_z\left[\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{1}{\rho}F_z+\nu\frac{1}{r}\frac{\partial V_z}{\partial r}\right]+$$

$$+3\chi.T^2\left[\left(\frac{\partial T}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial T}{\partial \varphi}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right]-\frac{\partial V_r}{\partial r}p-V_r\frac{\partial p}{\partial r}-\frac{\partial V_\varphi}{r\partial \varphi}p-\frac{V_\varphi}{r}\frac{\partial p}{\partial \varphi}-\frac{\partial V_z}{\partial z}p-V_z\frac{\partial p}{\partial z}$$

$$+\frac{\partial \nu}{\partial r}\left[2.V_r\frac{\partial V_r}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_r}{\partial \varphi}+V_\varphi\frac{\partial V_\varphi}{\partial r}-\frac{V_\varphi^2}{r}+V_z\frac{\partial V_r}{\partial z}+V_z\frac{\partial V_z}{\partial r}\right]+$$

$$+\frac{\partial \nu}{r\partial \varphi}\left[V_r\frac{1}{r}\frac{\partial V_r}{\partial \varphi}+V_r\frac{\partial V_\varphi}{\partial r}-\frac{V_rV_\varphi}{r}+2.V_\varphi\frac{1}{r}\frac{\partial V_\varphi}{\partial \varphi}+2\frac{V_rV_\varphi}{r}+V_z\frac{\partial V_\varphi}{\partial z}+\frac{V_z}{r}\frac{\partial V_z}{\partial \varphi}\right]+$$

$$+\frac{\partial \nu}{\partial z}\left[Vr\frac{\partial V_r}{\partial z}+V_r\frac{\partial V_z}{\partial r}+V_\varphi\frac{\partial V_\varphi}{\partial z}+V_\varphi\frac{1}{r}\frac{\partial V_z}{\partial \varphi}+2.V_z\frac{\partial V_z}{\partial z}\right]+$$

$$\left.+\nu\begin{bmatrix}2\left(\frac{\partial V_r}{\partial r}\right)^2+\frac{2}{r}\frac{\partial V_\varphi}{\partial r}\frac{\partial V_r}{\partial \varphi}+\left(\frac{\partial V_\varphi}{\partial r}\right)^2+\left(\frac{\partial V_z}{\partial r}\right)^2+3\frac{\partial V_r}{\partial z}\frac{\partial V_z}{\partial r}-2\frac{V_\varphi}{r}\frac{\partial V_\varphi}{\partial r}+\frac{1}{r^2}\left(\frac{\partial V_r}{\partial \varphi}\right)^2-\frac{V_\varphi}{r^2}\frac{\partial V_\varphi}{\partial \varphi}+\\ +\frac{2}{r^2}\left(\frac{\partial V_\varphi}{\partial \varphi}\right)^2+\frac{2}{r}V_\varphi\frac{\partial V_r}{\partial \varphi}+\frac{2}{r}V_r\frac{\partial V_\varphi}{\partial r}+\frac{2}{r}\frac{\partial V_\varphi}{\partial z}\frac{\partial V_z}{\partial \varphi}+\frac{1}{r}\left(\frac{\partial V_z}{\partial \varphi}\right)^2+2\left(\frac{\partial V_r}{\partial z}\right)^2+\left(\frac{\partial V_\varphi}{\partial z}\right)^2+2\left(\frac{\partial V_z}{\partial z}\right)^2\end{bmatrix}\right\}$$

We can neglect the second derivatives in the equation of motion too and the squares of derivatives. But this will be in the future.

Numerical method

To be able to compare the results with our 2D simulations (Dimittrova et.al 1990, 1997, 2002), we will use the same numerical method – large particle method (Belotsercovskiy 1985).

In our first works (Dimittrova, Filipov 1990) we prove that this method is very quick.

We build the model using C++ programming language.

Building this model, we search for each point the ρ, T and tree components for V(r, φ, z) as a function of time and some parameters.

Parameters are: M_1, M_2, R_{12}, T_{inf}, ρ_0, ν_0, λ_0, χ_0, a, b, c d, e and f. Here T_{inf} and ρ_0 are the temperature and density at the first Lagrangian point.

Altering these parameters, we will investigate haw they effect on the dynamics of the flow and will try to understand which of them are responsible for some physical processes.

The purpose

We begin building of 3D numerical model because of following easons:

1. ***To find more detailed picture of accretion flow.***
2. ***With a hope that we can find the origin of some structures formation.***
3. ***To confirm the role of physical parameters over the formation and dynamics of flow structures.***
4. ***Searching some origin of the instabilities.***
5. ***To understand whether or not 2D model is useful and where it is.***
6. ***To prove that angular momentum transfer is really as result of spiral structure formation.***

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