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Multiplex synchronization method at spatial positioning of mobile object

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1. Problem formulation

Multi-position radio-technical space-based navigation systems are of great interest lately. However, at present not all the stages of the flight can be provided with the help of these systems, including the landing of the aircraft where too great an accuracy is required.

This determines the extraordinary topical of the problem of developing algorithms for optimum navigation information processing, in which higher accuracy is achieved in comparison with the standard ones owing to the more complete extracting of information which is contained in the signals received from the emission sources. Such is the algorithm that makes use of the time delays separation method, which is also known as the method of additional variable [1].

The frame of reference O_0XYZ that is used here has origin O_0. The origin is fixed with respect to the Earth and it is attached to some point. The axis O_0Y is oriented upward in local vertical line, the axis O_0X is oriented toward the motion direction of the mobile object, and O_0Z is perpendicular to the plane XO_0Y. It is oriented toward the right side of the motion direction of the mobile object.

The signals emitted from four emission sources are observed on board the mobile object. The coordinates of the emission sources are known $X_k(t)=(x_k(t),y_k(t),z_k(t))$, $k=\overline{1,4}$.

The state vector $\lambda^T=(x,V_x,y,V_y,z,V_z,\Delta,V_\Delta)$ includes: the coordinates of the mobile object $-x,y,z$; Δ — the scale disagreement of the mobile object with respect to the system time; V_x,V_y,V_z,V_Δ — the respective velocities of x,y,z and Δ.

$\xi(t)$ is observed at the receiver's input on board the mobile object

(1) $$\xi(t)=s(t,\lambda(t))+n(t)$$

where $s(t,\lambda(t))=\sum_{k=1}^{4}s_k(t-T_k(\lambda))$ is an useful signal, which represents the sum of the signals $s_k(t-T_k(\lambda))$ from each emission source; $s_k(t-T_k(\lambda))=f_k(t-T_k(\lambda))\cos[\omega_0(t-T_k(\lambda))]$; $f_k(t-T_k(\lambda))$ is a radio-signal envelope from the k-th emission source; $\omega_0=2\pi f_0$ is a circular frequency of the high-frequency filling; $T_k(\lambda)=\tau_k(\mathbf{X})+\Delta$ is the arrival time of the signal from the k-th emission source; $\tau_k(\mathbf{X})=c^{-1}\left[(x-x_k)^2+(y-y_k)^2+(z-z_k)^2\right]^{1/2}$ is the real time of the signal delay from the k-th emission source; c is the light velocity; $n(t)$ is a white Gaussian noise with zero mean and the correlation function given by $M\{n(t)n(t+\tau)\}=\frac{N}{2}\delta(\tau)$; N is a one-sided spectral density of $n(t)$; $\delta(\tau)$ is the delta function.

The amplitudes of received signals from the different emission sources in the zone of radio-visibility are assumed to be equal.

The state vector $\boldsymbol{\lambda}$ can be described through Gaussian diffusion Markov process which satisfies the system of stochastic differential equations

(2) $$\dot{\boldsymbol{\lambda}}=\boldsymbol{F}\boldsymbol{\lambda}+\boldsymbol{n}_\lambda(t)$$

or

(3) $$\left|\begin{array}{l}\dot{x}=V_x\\ \dot{V}_x=-\alpha_x V_x+n_x(t)\\ \dot{y}=V_y\\ \dot{V}_y=-\alpha_y V_y+n_y(t)\\ \dot{z}=V_z\\ \dot{V}_z=-\alpha_z V_z+n_z(t)\\ \dot{\Delta}=V_\Delta\\ \dot{V}_\Delta=-\alpha_\Delta V_\Delta+n_\Delta(t)\end{array}\right.$$

where $n_x(t), n_y(t), n_z(t)$ and $n_\Delta(t)$ are independent white Gaussian noises with one-sided spectral densities N_x, N_y, N_z, N_Δ and with zero means

$$M\{n_x(t)\}=M\{n_y(t)\}=M\{n_z(t)\}=M\{n_\Delta(t)\}=0.$$

The matrices of the drift coefficients and the diffusion coefficients are

$$F=\begin{vmatrix}0&1&0&0&0&0&0&0\\0&-\alpha_x&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&-\alpha_y&0&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&-\alpha_z&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&-\alpha_\Delta\end{vmatrix}$$

$$N_\lambda=\begin{vmatrix}0&0&0&0&0&0&0&0\\0&\frac{N_x}{2}&0&0&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&\frac{N_y}{2}&0&0&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&\frac{N_z}{2}&0&0\\0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&\frac{N_\Delta}{2}\end{vmatrix}.$$

The noise vector is $n_\lambda^T(t)=[0,n_x(t),0,n_y(t),0,n_z(t),0,n_\Delta(t)]$ with correlation matrix $M\{n_\lambda(t)n_\lambda^T(t-\tau)\}=N_\lambda\delta(\tau)$.

The essence of the multiplex synchronization method (MSM) is displayed in [1—4].

2. Optimal filtration algorithm based upon a multiplex synchronization

An additional variables vector $T_d^T=(T_{d1},T_{d2},T_{d3},T_{d4})$ is introduced for the signals from the four emission sources. T_{d1},T_{d2},T_{d3} and T_{d4} are the time delays of the high frequency filling of the received signals from the separate emission sources respectively. T is a transposition symbol. Then the state vector λ is expanded to the new state vector $\lambda_d^T=\{\lambda^T,T_d^T\}$ [1].

The filtration with grouping of observations at the step-function approximation is a very suitable algorithm. In this case the dependence of the signal $s_k(t-T_k(\lambda))$ on the comparatively slowly varying processes $x(t), y(t), z(t)$ and $\Delta(t)$ must be considered. These processes are approximately invariable for the sampling interval (T). Therefore, we can approximate the observation in the frame of each sampling interval $(t_\nu, t_{\nu+1})$ by the equation

$$\xi(t) = s(t, \lambda_\nu) + n(t), t \in (t_\nu, t_{\nu+1}), \nu = 1,2,3,...$$

where $\lambda_\nu = \lambda(t_\nu)$ is a value of the parameter in the supporting points $t_\nu = \nu t$.

The sequence λ_ν satisfies the recursion equation

(4) $$\lambda_{\nu+1} = \Phi\lambda_\nu + n_{\lambda_\nu}$$

where $\Phi = \exp\{FT\}$ is the state transition matrix for sampling interval (T); n_{λ_ν} is a sequence of independent Gaussian random vectors with zero mean $M\{n_{\lambda_\nu}\} = 0$ and correlation matrix $\Psi = M\{n_{\lambda_\nu}\ n_{\lambda_\nu}^T\}$.

When the method of additional variable is used, the observation equation is given by equations [1—4]

(5) $$\xi(t) = s(t, \lambda_{d\nu}) + n(t), \ t \in (t_\nu, t_{\nu+1}),$$

(6) $$s(t, \lambda_\nu, T_{d\nu}) = \sum_{k=1}^{4} f_k[t - T_k(\lambda_\nu)]\cos[\omega_0(t - T_{dk\nu})]$$

where $\lambda_{d\nu}^T = \{\lambda_\nu^T, T_{d\nu}^T\}; T_{d\nu} = T_d(t_\nu); t_\nu = \nu T$.

The equation that expresses the dynamic behaviour of the system has the form

(7) $$\lambda_{d(\nu+1)} = \Phi_d\ \lambda_{d\nu} + n_{\lambda_{d\nu}}$$

where $\Phi_d = \exp\{F_d T\}$ is the state transition matrix for sampling interval (T), when the state vector is expanded with $T_d^T = (T_{d1}, T_{d2}, T_{d3}, T_{d4})$; F_d is a matrix of the drift coefficients; $n_{\lambda_{d\nu}}$ is a sequence of independent Gaussian random vectors with zero mean $M\{n_{\lambda_{d\nu}}\} = 0$ and correlation matrix

$$\Psi_d = M\{n_{\lambda_d\ \nu}, n_{\lambda_d\ \nu}^T\} = \int_0^T \exp\{F_d(T-\tau)\} N_{\lambda d} \exp\{F_d^T(T-\tau)\} d\tau.$$

The matrices of the drift coefficients and the diffusion coefficients for expanding the state vector are given by [1]:

$$F_d = \begin{vmatrix} F & O_{(8\times4)} \\ F_{T_d} & O_{(4\times4)} \end{vmatrix}; \ N_{\lambda_d} = \begin{vmatrix} N_\lambda & O_{(8\times4)} \\ O_{(4\times8)} & O_{(4\times4)} \end{vmatrix},$$

where $O_{(m\times n)}$ is a zero matrix with dimension $m\times n$;

$$F_{T_d} = \begin{vmatrix} 0 & c^{-1}\cos\alpha_1 & 0 & c^{-1}\cos\beta_1 & 0 & c^{-1}\cos\gamma_1 & 0 & 1 \\ 0 & c^{-1}\cos\alpha_2 & 0 & c^{-1}\cos\beta_2 & 0 & c^{-1}\cos\gamma_2 & 0 & 1 \\ 0 & c^{-1}\cos\alpha_3 & 0 & c^{-1}\cos\beta_3 & 0 & c^{-1}\cos\gamma_3 & 0 & 1 \\ 0 & c^{-1}\cos\alpha_4 & 0 & c^{-1}\cos\beta_4 & 0 & c^{-1}\cos\gamma_4 & 0 & 1 \end{vmatrix}$$

$\cos\alpha_k, \cos\beta_k, \cos\gamma_k$ are direction cosines of the k-th emission source.

The expressions of the matrices and may be represented as follows

$$
(8) \qquad \Phi_d = \begin{vmatrix}
1 & \Phi_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \Phi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \Phi_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \Phi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \Phi_{56} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \Phi_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \Phi_{78} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{88} & 0 & 0 & 0 & 0 \\
0 & \Phi_{9,2} & 0 & \Phi_{9,4} & 0 & \Phi_{9,6} & 0 & \Phi_{9,8} & 1 & 0 & 0 & 0 \\
0 & \Phi_{10,2} & 0 & \Phi_{10,4} & 0 & \Phi_{10,6} & 0 & \Phi_{10,8} & 0 & 1 & 0 & 0 \\
0 & \Phi_{11,2} & 0 & \Phi_{11,4} & 0 & \Phi_{11,6} & 0 & \Phi_{11,8} & 0 & 0 & 1 & 0 \\
0 & \Phi_{12,2} & 0 & \Phi_{12,4} & 0 & \Phi_{12,6} & 0 & \Phi_{12,8} & 0 & 0 & 0 & 1
\end{vmatrix},
$$

where

$$\Phi_{12} = \frac{1-e^{-\alpha_x T}}{\alpha_x}; \ \Phi_{34} = \frac{1-e^{-\alpha_y T}}{\alpha_y}; \ \Phi_{56} = \frac{1-e^{-\alpha_z T}}{\alpha_z}; \ \Phi_{78} = \frac{1-e^{-\alpha_\Delta T}}{\alpha_\Delta};$$

$$\Phi_{22} = e^{-\alpha_x T}; \ \Phi_{44} = e^{-\alpha_y T}; \ \Phi_{66} = e^{-\alpha_z T}; \ \Phi_{88} = e^{-\alpha_\Delta T};$$

$$\Phi_{9,2} = \frac{\cos\alpha_1}{c}\Phi_{12}; \Phi_{9,4} = \frac{\cos\beta_1}{c}\Phi_{34}; \ \Phi_{9,6} = \frac{\cos\gamma_1}{c}\Phi_{56}; \ \Phi_{9,8} = \Phi_{78};$$

$$\Phi_{10,2} = \frac{\cos\alpha_2}{c}\Phi_{12}; \Phi_{10,4} = \frac{\cos\beta_2}{c}\Phi_{34}; \ \Phi_{10,6} = \frac{\cos\gamma_2}{c}\Phi_{56}; \ \Phi_{10,8} = \Phi_{78};$$

$$\Phi_{11,2} = \frac{\cos\alpha_3}{c}\Phi_{12}; \Phi_{11,4} = \frac{\cos\beta_3}{c}\Phi_{34}; \ \Phi_{11,6} = \frac{\cos\gamma_3}{c}\Phi_{56}; \ \Phi_{11,8} = \Phi_{78};$$

$$\Phi_{12,2} = \frac{\cos\alpha_4}{c}\Phi_{12}; \Phi_{12,4} = \frac{\cos\beta_4}{c}\Phi_{34}; \ \Phi_{12,6} = \frac{\cos\gamma_4}{c}\Phi_{56}; \ \Phi_{12,8} = \Phi_{78};$$

(9)

$$\Psi_d = \begin{vmatrix}
\Psi_{11} & \Psi_{12} & 0 & 0 & 0 & 0 & 0 & 0 & \Psi_{1,9} & \Psi_{1,10} & \Psi_{1,11} & \Psi_{1,12} \\
\Psi_{21} & \Psi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & \Psi_{2,9} & \Psi_{2,10} & \Psi_{2,11} & \Psi_{2,12} \\
0 & 0 & \Psi_{33} & \Psi_{34} & 0 & 0 & 0 & 0 & \Psi_{3,9} & \Psi_{3,10} & \Psi_{3,11} & \Psi_{3,12} \\
0 & 0 & \Psi_{43} & \Psi_{44} & 0 & 0 & 0 & 0 & \Psi_{4,9} & \Psi_{4,10} & \Psi_{4,11} & \Psi_{4,12} \\
0 & 0 & 0 & 0 & \Psi_{55} & \Psi_{56} & 0 & 0 & \Psi_{5,9} & \Psi_{5,10} & \Psi_{5,11} & \Psi_{5,12} \\
0 & 0 & 0 & 0 & \Psi_{65} & \Psi_{66} & 0 & 0 & \Psi_{6,9} & \Psi_{6,10} & \Psi_{6,11} & \Psi_{6,12} \\
0 & 0 & 0 & 0 & 0 & 0 & \Psi_{77} & \Psi_{78} & \Psi_{7,9} & \Psi_{7,10} & \Psi_{7,11} & \Psi_{7,12} \\
0 & 0 & 0 & 0 & 0 & 0 & \Psi_{87} & \Psi_{88} & \Psi_{8,9} & \Psi_{8,10} & \Psi_{8,11} & \Psi_{8,12} \\
\Psi_{9,1} & \Psi_{9,2} & \Psi_{9,3} & \Psi_{9,4} & \Psi_{9,5} & \Psi_{9,6} & \Psi_{9,7} & \Psi_{9,8} & \Psi_{9,9} & \Psi_{9,10} & \Psi_{9,11} & \Psi_{9,12} \\
\Psi_{10,1} & \Psi_{10,2} & \Psi_{10,3} & \Psi_{10,4} & \Psi_{10,5} & \Psi_{10,6} & \Psi_{10,7} & \Psi_{10,8} & \Psi_{10,9} & \Psi_{10,10} & \Psi_{10,11} & \Psi_{10,12} \\
\Psi_{11,1} & \Psi_{11,2} & \Psi_{11,3} & \Psi_{11,4} & \Psi_{11,5} & \Psi_{11,6} & \Psi_{11,7} & \Psi_{11,8} & \Psi_{11,9} & \Psi_{11,10} & \Psi_{11,11} & \Psi_{11,12} \\
\Psi_{12,1} & \Psi_{12,2} & \Psi_{12,3} & \Psi_{12,4} & \Psi_{12,5} & \Psi_{12,6} & \Psi_{12,7} & \Psi_{12,8} & \Psi_{12,9} & \Psi_{12,10} & \Psi_{12,11} & \Psi_{12,12}
\end{vmatrix},$$

where

$$\Psi_{11} = \frac{N_x}{4\alpha_x^3}\left(4e^{-\alpha_x T} - e^{-2\alpha_x T} + 2\alpha_x T - 3\right);\ \Psi_{12} = \frac{N_x}{4\alpha_x^2}\left(e^{-2\alpha_x T} - 2e^{-\alpha_x T} + 1\right);$$

$$\Psi_{22} = \frac{N_x}{4\alpha_x}\left(1 - e^{-2\alpha_x T}\right);\quad \Psi_{33} = \frac{N_y}{4\alpha_y^3}\left(4e^{-\alpha_y T} - e^{-2\alpha_y T} + 2\alpha_y T - 3\right);$$

$$\Psi_{34} = \frac{N_y}{4\alpha_y^2}\left(e^{-2\alpha_y T} - 2e^{-\alpha_y T} + 1\right);\ \Psi_{44} = \frac{N_y}{4\alpha_y}\left(1 - e^{-2\alpha_y T}\right);$$

$$\Psi_{55} = \frac{N_z}{4\alpha_z^3}\left(4e^{-\alpha_z T} - e^{-2\alpha_z T} + 2\alpha_z T - 3\right);\ \Psi_{56} = \frac{N_z}{4\alpha_z^2}\left(e^{-2\alpha_z T} - 2e^{-\alpha_z T} + 1\right);$$

$$\Psi_{66} = \frac{N_z}{4\alpha_z}\left(1 - e^{-2\alpha_z T}\right);\ \Psi_{77} = \frac{N_\Delta}{4\alpha_\Delta^3}\left(4e^{-\alpha_\Delta T} - e^{-2\alpha_\Delta T} + 2\alpha_\Delta T - 3\right);$$

$$\Psi_{78} = \frac{N_\Delta}{4\alpha_\Delta^2}\left(e^{-2\alpha_\Delta T} - 2e^{-\alpha_\Delta T} + 1\right);\ \Psi_{88} = \frac{N_\Delta}{4\alpha_\Delta}\left(1 - e^{-2\alpha_\Delta T}\right);$$

$$\Psi_{1(k+8)}=\frac{\cos\alpha_k}{c}\Psi_{11};\ \Psi_{2(k+8)}=\frac{\cos\alpha_k}{c}\Psi_{12};\ \Psi_{3(k+8)}=\frac{\cos\beta_k}{c}\Psi_{33};$$

$$\Psi_{4(k+8)}=\frac{\cos\beta_k}{c}\Psi_{34};\ \Psi_{5(k+8)}=\frac{\cos\gamma_k}{c}\Psi_{55};\ \Psi_{6(k+8)}=\frac{\cos\gamma_k}{c}\Psi_{56};$$

$$\Psi_{7(k+8)}=\Psi_{77};\quad \Psi_{8(k+8)}=\Psi_{78};$$

$$\Psi_{(k+8)(l+8)}=\frac{\cos\alpha_k}{c}\frac{\cos\alpha_l}{c}\Psi_{11}+\frac{\cos\beta_k}{c}\frac{\cos\beta_l}{c}\Psi_{33}+\frac{\cos\gamma_k}{c}\frac{\cos\gamma_l}{c}\Psi_{55}+\Psi_{77};$$

$$\Psi_{ij}=\Psi_{ji};\ \left(k,l=\overline{1,4}\right);\ \left(i,j=\overline{1,12}\right).$$

When the fact, that $\boldsymbol{\lambda}_d$ is a vector of non-power parameters, is taken into account, the expanded Kalman filter state equation and the equation of covariance matrix of the filtered errors $\boldsymbol{R}_\nu$ can be represented by following equation [1, 4]

$$\hat{\boldsymbol{\lambda}}_{d(\nu+1)}=\tilde{\boldsymbol{\lambda}}_{d(\nu+1)}+\frac{2}{N}\boldsymbol{R}_{\nu+1}\int_{t_\nu}^{t_{\nu+1}}\xi(t)\frac{\partial s\left(t,\tilde{\boldsymbol{\lambda}}_{d(\nu+1)}\right)}{\partial\boldsymbol{\lambda}_d}dt; \tag{10}$$

$$\boldsymbol{R}_{\nu+1}^{-1}=\tilde{\boldsymbol{R}}_{\nu+1}^{-1}+\frac{2}{N}\int_{t_\nu}^{t_{\nu+1}}\left[\frac{\partial s\left(t,\tilde{\boldsymbol{\lambda}}_{d(\nu+1)}\right)}{\partial\boldsymbol{\lambda}_d^T}\right]^T\left[\frac{\partial s\left(t,\tilde{\boldsymbol{\lambda}}_{d(\nu+1)}\right)}{\partial\boldsymbol{\lambda}_d^T}\right]dt, \tag{11}$$

where

$\tilde{\boldsymbol{\lambda}}_{d(\nu+1)}=\boldsymbol{\Phi}_d\hat{\boldsymbol{\lambda}}_{d\nu}$ is a vector of the state prediction;

$\tilde{\mathrm{R}}_{\nu+1}=\Phi_d\boldsymbol{R}_\nu\Phi_d^T+\Psi_{d\nu}$ is a prediction-error covariance matrix;

$$\hat{\boldsymbol{\lambda}}_{d\nu}^T=\left\{\hat{\boldsymbol{\lambda}}_\nu^T,\hat{\boldsymbol{T}}_{d\nu}^T\right\};\ \hat{\boldsymbol{\lambda}}_\nu^T=\left\{\hat{x}_\nu,\hat{V}_{x\nu},\hat{y}_\nu,\hat{V}_{y\nu},\hat{z}_\nu,\hat{V}_{z\nu},\hat{\Delta}_\nu,\hat{V}_{\Delta\nu}\right\};$$

$$\hat{\boldsymbol{T}}_{d\nu}^T=\left\{\hat{T}_{d1\nu},\hat{T}_{d2\nu},\hat{T}_{d3\nu},\hat{T}_{d4\nu}\right\};\ \boldsymbol{R}_\nu=\begin{vmatrix}\boldsymbol{R}_{\lambda_\nu} & \boldsymbol{R}_{\lambda T_{d\nu}}\\ \boldsymbol{R}_{T_d\lambda_\nu} & \boldsymbol{R}_{T_{d\nu}}\end{vmatrix};\ \boldsymbol{R}_{\lambda T_{d\nu}}=\boldsymbol{R}_{T_d\lambda_\nu}^T.$$

The maximum likelihood estimate of the vector of the corrected time delays of the signals from the separate emission sources is [1]

$$\boldsymbol{T}_\nu^*\approx\min_k{}^{-1}\left|\hat{\boldsymbol{T}}_{d\nu}+\boldsymbol{k}_\nu T_0-\boldsymbol{T}\left(t,\hat{x}_\nu,\hat{y}_\nu,\hat{z}_\nu,\hat{\Delta}_\nu\right)\right|=\hat{\boldsymbol{T}}_{d\nu}+\boldsymbol{k}_\nu^*T_0, \tag{12}$$

where $\left(k_\nu^*\right)^T=\left[k_{1\nu}^*,k_{2\nu}^*,k_{3\nu}^*,k_{4\nu}^*\right]$ is a vector of values of $\left(k_\nu\right)^T=\left[k_{1\nu},k_{2\nu},k_{3\nu},k_{4\nu}\right]$, at which an extremum is reached; $k_{i\nu}=1,2,3,...;\ i=\overline{1,4};\ \left(T_\nu^*\right)^T=\left[T_{1\nu}^*,T_{2\nu}^*,T_{3\nu}^*,T_{4\nu}^*\right]$ is a corrected time delays vector of the signals from the separate emission sources.

So, the problem is reduced to new one supposing resolving the following equation system

$$
(13)\quad \begin{cases}
T_{1\nu}^* = \hat{T}_{d1\nu} + k_{1\nu}^* T_0 = c^{-1}\left[\left(x_\nu^* - x_1\right)^2 + \left(y_\nu^* - y_1\right)^2 + \left(z_\nu^* - z_1\right)^2\right]^{1/2} + \Delta_\nu^* \\
T_{2\nu}^* = \hat{T}_{d2\nu} + k_{2\nu}^* T_0 = c^{-1}\left[\left(x_\nu^* - x_2\right)^2 + \left(y_\nu^* - y_2\right)^2 + \left(z_\nu^* - z_2\right)^2\right]^{1/2} + \Delta_\nu^* \\
T_{3\nu}^* = \hat{T}_{d3\nu} + k_{3\nu}^* T_0 = c^{-1}\left[\left(x_\nu^* - x_3\right)^2 + \left(y_\nu^* - y_3\right)^2 + \left(z_\nu^* - z_3\right)^2\right]^{1/2} + \Delta_\nu^* \\
T_{4\nu}^* = \hat{T}_{d4\nu} + k_{4\nu}^* T_0 = c^{-1}\left[\left(x_\nu^* - x_4\right)^2 + \left(y_\nu^* - y_4\right)^2 + \left(z_\nu^* - z_4\right)^2\right]^{1/2} + \Delta_\nu^*,
\end{cases}
$$

at which the maximum likelihood estimates $x_\nu^*, y_\nu^*, z_\nu^*$ and Δ_ν^* are determined.

3. Algorithm accuracy

Fig. 1 represents results of the research of the accuracy characteristics when positioning a mobile object both with and without making use of the multiplex synchronization. The research has been carried out for the NAVSTAR satellite navigation system for Geometric Dilution of Precision GDOP=4 for C/A- and P-code. The computations have been carried out for typical values of the parameters: signal noise ratio $q=1\div100$; sampling interval $T = 0{,}1$ s; standard deviation of velocity $\sigma_V = 1\text{m/s}$; frequency band of the noise in the motion model of the mobile object $\alpha = 0{,}1\text{Hz}$; parameters of the signal (1): carrier frequency $f_0 = \omega_0/2\pi = 1575{,}42\text{MHz}$; continuance of one element of the range-finder code: for C/A-code $\tau_e = 980\text{ns}$; for P-code $\tau_e = 98\text{ns}$; initial values of the standard deviation of the estimate parameters: $\sigma_{x0} = 300\text{m}$; $\sigma_{V_{x0}} = 300\text{m/s}$.

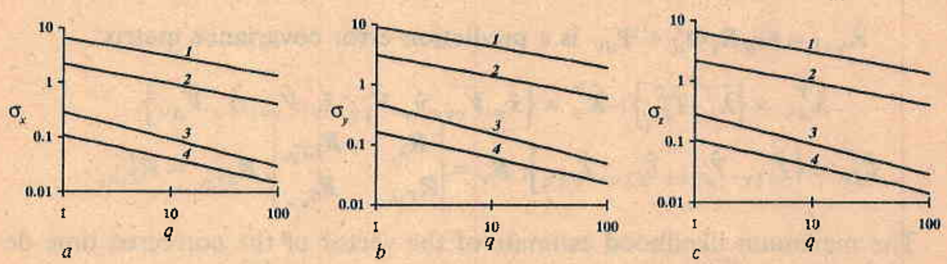

Fig. 1. Standard deviation when determining the coordinates of the mobile object

a — for *x*; *b* — for *y*; *c* — for *z*; *1* — without using MSM for C/A-code; *2* — without using MSM for P-code; *3* — using MSM for C/A-code; *4* — using MSM for P-code

The displayed diagrams represent the standard deviation when determining the coordinates 1 hour after the beginning of the work of the algorithm. When applying the multiplex synchronization method for this time the fluctuation error, using the C/A-code turns out to be 40÷45 times smaller compared with the standard algorithm one, whereas using the P-code it is 25÷30 times smaller.

Table 1 represents the horizontal and vertical mean square error of a mobile object positioning for the signal noise ratio $q=100$ when applying the multiplex synchronization method and using C/A-code. The displayed data show the superiority of the multiplex synchronization method in the accuracy characteristics in comparison with the standard method for filtration.

Table 1

Time, h	1	6	12	18	24	36	Stationary mode
Horizontal error, mm	29	14	11	10	9	9	8
Vertical error, mm	48	22	17	16	15	14	12

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Метод на обединената синхронизация при пространствено позициониране на подвижен обект

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(Резюме)

Приложен е методът на обединената синхронизация в задачата за трикоординатно позициониране на подвижен обект (ПО) чрез използване на многопозиционна радионавигационна система. При този метод се следят времезакъсненията на далекомерния код и високочестотното запълнение. След това по тези данни се извършва оценка на времезакъснението на приетия сигнал по метода на максималното правдоподобие. На базата на получените резултати се изчисляват координатите на ПО. Рекурентният алгоритъм за обработка на приетите радионавигационни сигнали е разработен на основата на оптималната филтрация. Определянето на местоположението на ПО се извършва по квазидалекомерен метод, при който се отчита разсъгласуването на еталонния генератор на борда на ПО спрямо системното време.