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Self-organization theory and self-organization prbolems of magnetohydrodynamical accretion disc theory with reference to the process of large-scale turbulent helicity structure formation in accretion discs around black holes and neutron stars

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I. Introduction

During the past 20 years there has been an apparent interest in some complex phenomena in physics and other natural sciences such as the formation of spatial structures in hydrodynamical flows (Bernard's cells in thermoconvection, Taylor vortices between rotating cylinders), distributions of populations in ecological systems, nonequilibrium chemical reactions and many others. Although various in their nature, all of them can be described by means of similar models and equations in the framework of the so-called self-organization or synergetic theory [1-4]. As a rule, self-organization can be regarded as an appearence of spatial (generally, evolving with time) structures in a dissipative, nonequilibrium medium. Moreover, the parameters, characterizing these structures are determined by the properties of the medium itself and not by the spatial structure of the nonequilibrium source (energy, mass and etc.) [5].

The present paper has the purpose to present the basic theoretical principles, on which a consistent self-organization theory of magnetohydrodynamical accretion discs around compact astrophysical objects (black holes and neutron stars) can be created. Such a theory requires to regard accretion discs as open, nonequilibrium and dissipative systems. The main reasons for such a treatment are obvious. For example, an accretion disc is an open system due to the following reasons: 1) there is an input of accreting matter from the outside companion compact object (a massive star) with an accretion rate M, which is an important parameter in view of observable effects such as spectra, bursts and etc. 2) the gravitational field of the compact object sets up the vertical density distribution in the accretion disc in dependence of the variations of the gravitational potential [6]. 3) the magnetic field of a neutron star strongly influences the movement of matter, especially in the boundary layer of the accretion disc around the neutron star.

Since in accretion discs processes of substantial energy release occur due to viscous dissipation, reconnection of magnetic field lines and radiation transfer, accretion discs can also be regarded as typical dissipative systems. Also, since the unsteady accretion rate M is connected with the complex time variations in the X-ray spectra of accretion discs, they can be considered as nonequilibrium systems.

In the next chapter the specific process of turbulent magnetohydrodynamical helicity structures formation will be investigated in the framework of self-organization theory.

II. Basic principles of the self-organization approach in magnetohydrodynamical accretion disc theory

The first important principle is that of hierarchy, which means that the initially formed turbulent structures (in the case-vortices) can interact between each other to form a new, hierarchical system of structures-large-scale helicity structures. Such a process, occuring in the boundary layers of acretion discs around neutron stars, has already been investigated in reference [7]. However, the interaction and the exchange of energy between the vortices and the subsequent deviation from isotropic turbulence give some reasons to assert in this paper that this process of anisotropization of turbulence should be accounted by additional terms in the spectral energy transfer equation:

$$\frac{\partial E}{\partial t}=-2\nu k^2 E\ (k,\ t)+T\ (k,\ t), \tag{1}$$

where $E\ (k,\ t)$ is the nonstationary energy function in k representation (k-wave number), $T\ (k,\ t)$ is the spectral transfer function in the inertial range and ν is the turbulent viscosity. The additional terms in (1) due to the interaction between the vortices depend more especially on the interaction function [8], which can account for the process of "energy accumulation" in the small wave numbers, characteristic for the case of two-dimensional turbulence. Moreover, since in accretion discs around neutron stars the twisting and the reconnection of magnetic field lines limits the energy cascade towards larger space scales (i. e. small k), the energy cascade in two opposite directions should be taken into consideration:

a) in the direction of small k, accounting for the orientation of two-dimensional vortices in the direction of the magnetic field;

b) in the direction of large k, thus accounting for the disorientating (chaotical) action of three-dimensional turbulence [9].

The second important principle of the self-organization approach concerns the nonequilibrium energy exchange in accretion discs. The process of stationary energy transfer from gaseous turbulence to the magnetic field has already been investigated in references [10-12]. However, in view of the basic principles of structure formation in self-organization theory it is natural to expect that the entropy of the open dissipative system "gaseous turbulence-magnetic field" will decrease in the process of helicity structure formation. In other words, this means that the assumption about isotropic magnetic diffusion and reconnection should break down and therefore, the system of equations, describing the magnetic field generation in a geometrically thin disc will no longer be valid.

The third important principle is about the establishment or breaking of a given type of symmetry of the magnetic field. On the base of the invariance of the system of equations (describing the magnetic field generation) under the transformation $z \to -z$ it has been proved in ref. [13] that two types of solutions of this system can be distinguished, corresponding to two different modes and symmetries of the magnetic field:

Ist type of symmetry is

$$B_r(z) = -B_r(-z); \; B_\varphi(z) = -B_\varphi(-z); \; B_z(z) = B_z(-z) \tag{2}$$

for the dipole mode and

$$B_r(z) = B_r(-z); \; B_\varphi(z) = B_\varphi(-z); \; B_z(z) = -B_z(-z) \tag{3}$$

for the quadrupole mode. However, when anisotropic diffusion sets in and under the assumption that the fluctuating field's scale is less than the characterisctic scale of the mean field, the fluctuating component of the expression $\varepsilon_i = \overline{v' x b'}$ in the above system of equations should be taken of the kind:

$$\varepsilon_i = \overline{v' x b'} = \alpha_{ij} B_i + \beta_{ijk} \frac{\partial B_k}{\partial x_i} \tag{4}$$

$i, j, k = r, \varphi, z.$; α_{ij} is the helicity tensor, β_{ijk} is the anisotropic tensor of magnetic diffusion, B_i is the mean magnetic field and v' and b' are the fluctuating components of the turbulent velocity and of the magnetic field correspondingly. It is evident that the symmetry in this case will be different and more complex and will not be connected only with the z coordinate. It should be noted that it is not clear what type will be the symmetry at the final stage of the development of the helicity structure, when the scales of the fluctuating and of the mean field become comparable and equation (4) is inapplicable.

The fourth important principle concerns the connection between the helicity structure formation and the changes in the turbulent viscosity as a parameter, characterizing the turbulent medium, in accordance with what has been mentioned in the Introduction of this paper. In reference [10] the total viscosity had been presented as a sum of the viscosity when $k < k_1$ and of the "anomalous" viscosity (when $k > k_1$), where k_1 is the wave number when anomalous viscosity sets in. At the same time the spectral function $F(k)$ in the right-hand side of the correlation equation:

$$\frac{1}{2} \langle v_r(k) \, v_r(-k) \rangle = \frac{1}{3} \frac{F(k)}{4\pi k^2} \tag{5}$$

(the brackets $\langle \ldots \rangle$ denote spatial or time averaging) is typical for the isotropic turbulence. At this important point the following remark is made in this paper: the clear distinction in ref. [10] between the two types of viscosities (when $k < k_1$ and $k > k_1$) is physically inacceptible and idealized assumption.

The reason is that "anomalous"vortices "overlap" the isotropical turbulence and hence a correlation in the helicity due to the presence of the term

$$-\Phi = \int_0^\infty \langle \alpha(t)\, \alpha(t+s) \rangle\, ds; \ \alpha - \text{helicity} \tag{6}$$

in the diffusion term in the system of equations, describing the generation of magnetic field. That is why it is proposed in this paper that another, antisymmetrical term should be added to the correlation function $F_y(k, t)$:

$$F_{ij}(k,t) = \frac{E(k,t)}{4\pi k^2} (k^2 \delta_{ij} - k_i k_j) + \frac{i K(k,t)}{8\pi k^4} \varepsilon_{ijf} k_f , \tag{7}$$

where the spectral functions $E(k, t)$ and $K(k, t)$ are connected with the turbulent velocity by the following formulae:

$$\frac{1}{2} \langle v^2 \rangle = \int_0^\infty E(k, t)\, dk; \ \langle v, \mathrm{rot}\, v \rangle = \int_0^\infty K(k,t)\, dk. \tag{8}$$

It should be noted also that no explanation has yet been proposed of the following facts, connected with anomalous diffusion:

a) what is the physical meaning of the change of sign in the dipole (oscillating) mode of the magnetic field and is this change related with the exitence of anomalous diffusion?

b) it is known that anomalous diffusion is connected with the appearence of large-scale, long-lived vortices. However, it is not known presently whether large-scale helicity structures should always be caused by anomalous diffusion?

The fifth important principle of the self-organization approach concerns the application of the methods of nonlinear physics. In fact the assumption about nonlinearity is in full accordance with the previous assumption about the occurence of dissipative processes. To prove the last statement let us assume the contrary. But then from Liuvile' theorem about the constancy of phase volume for ergodic systems (i. e. for systems without dissipation) it will follow that when $t \to \infty$ the correlation function

$$\langle v_\alpha(k, t)\, v_\beta(k, t) \rangle \neq 0 \tag{9}$$

and also

$$\langle v_\alpha(k, t)\, v_\beta(p, t)\, v_\gamma(-k-p, t) \rangle = 0, \ [14]. \tag{10}$$

The last equality in practice excludes the three-mode interactions, which is highly improbable, especially at the later-time process of interaction between the vortices. Moreover, in a turbulent medium with nonlinearity and dispersion (due to the viscosity) a nonlinear interaction between the perturbations along the r, φ, and z directions will take place, resulting in either competition or synchronization between the different modes and also processes of self-focusing or modulations, similarly to many known processes in radiophysics and nonlinear wave propagation theory [15]. Therefore, the application of such nonlinear approaches suggests that the existing linear theory of disc instabilities should be seriously revisited. In the framework of a new nonlinear disc instabilities theory, the stretching of convective cells along the z direction, also the growth of nonaxisymmetrical azimutal perturbations, discussed in ref.

[16], probably would be explained with the effect of competition between the modes.

The above proposed five guiding principles allow one to make the conclusion that the variations of density, turbulent velocity, magnetic field, intensity of radiation and etc. would depend on the concrete physical parameters, characterizing the accretion disc such as the accretion rate M, the parameter $\beta=p_r/(p_r+p_g)$ (p_r — radiation pressure, p_g — gas pressure), the magnetic Reynold's number $R_m=vl/\nu_m$(l — characteristic length of the magnetic field, ν_m — the magnetic viscosity), the dynamo number $D=(R\frac{\partial w}{\partial R})\alpha_0 z_0^3/\nu_m^3$ (R— disc radius, w — angular rotational velocity, l — characteristic length along z direction) and also the parameter δ, defined by the formulae:

$$\frac{\rho\langle v^2\rangle}{2}\delta=\frac{\rho\langle v^2\rangle}{2}-\frac{\langle H^2\rangle}{8\pi}\quad(0<\delta<1). \tag{11}$$

From (11) it is evident that δ characterizes the rate of pulsational kinetic (i. e. turbulent) and magnetic energy and therefore the degree of nonequilibricity of the system "gaseous turbulence — magnetic field". One of the most important consequences of the application of the self-organization approach in this paper is that this "nonequilibricity" can be quantitatively characterized in terms of the other parameters of the accretion disc. This is natural to be expected since in self-organization theory it is important to determine which are the "governing" and the "submitted" parameters [3, 4]. This is not an easy task and is not always possible. However, a more valuable idea can be suggested — the stohastic analyses of the intensity fluctuations of radiation and the quantitative characterization of the level of stohasticity through the dimensions of the stohastic attractor [17], can give valuable information about the magnitude of the magnetic Reynold's number and many other parameters.

The next chapter will be devoted to some observational facts from black hole and neutron star physics, which also suggest the idea about structure formation in accretion discs.

III. Observational facts in support of the large-scale turbulent helicity structure formation in accretion discs

The profound analyses of the X-ray spectra shows that although they are varied, they are also in a certain sense "symmetric" and "deterministic" on a macro and on a "micro" scale as well. Some observational data in support of this hypothesis and reflecting also the process of accretion are the following:

1. The existance of a "white noise" (i. e. correlation in the impulses) in the X-ray spectrum of Her X-1 due to the influence of the accretion disc.
2. In some X-ray sources (such as Her X-1 and Cen X-3) the main profile of the impulses does not change for a great range of energies, while for other sources (441626-67) this profile depends on energy.
3. Age decrease of the period of repetition of the impulses and therefore age increase of the rotation frequency of neutron stars in binary systems.
4. The great variety of the impulse forms (from symmetrical to highly asymmetrical).

If the "macro" spectrum reflects physical processes under the influence of outer factors (for example the accretion rate), then the "micro" spectrum reflects the ability of the system (in the case — the accretion disc) to create its "own signals" as a response to the action of the outer factors. That is why a conclusion is made in this paper that the "micro" spectrum should reflect a more complex dependence of the physical phenomena and parameters.

In this chapter two important and yet unresolved problems will be considered, which are based on observational data and are supposed to be related to the process of large-scale helicity structure formation.

The first problem is connected with the decrease and the subsequent increase of the period of pulsations in some X-ray sources for a given period of time (source 440900-40 for periods of 4 and 2 years respectively). A possible explanation for this is the change of the direction of rotation of the accreting matter near the magnetosphere, proposed in ref. [18]. Besides, in certain X-ray sources this change of period (transition "spin up" — "spin down") takes place for short times, while for others (Vela X-1) it occurs for longer periods (50 years). It can naturally be assumed that the reason for such a transition is connected with the structurization of the accretion flow.

The second problem concerns the existence of a "soft" (with small energy) and of a "hard" (with greater energy) components in the X-ray radiation from neutron stars in binary systems and also from "possible" candidates for black holes (CYG X-1, CIR X-1 and others). In the second case of a black hole the existence of a "high" state ("soft" component with energy $E<10$ keV) is explained by means of the theoretical assumption about a higher accretion rate ($\dot{M}>M_{cr}$), while the "low state" (with a "hard" component of radiation) is connected with the decrease of the accretion rate below a certain critical value $\dot{M}_{cr}$, i. e. $\dot{M}<\dot{M}_{cr}$. It is supposed also that the observed low temperature spectrum during the "low" state is connected with the existence of a coronae around the disk and with the increased activity of the magnetic field. The following observational fact is very important: the increase of the intensity of radiation (approximately several times, without any change in the spectrum) at the end of the "high" state. This clearly suggests that the transition "high-low" state takes place gradually and the prerequisites for the transition to the "low" state for example appear yet during the "high" state. Evidently there is a "transition time", during which structural changes in the accretion disc may take place, including the formation of helicity structures.

As far as accretion discs around neutron stars are concerned, the "soft" component of radiation is relatively stable in intensity and is identified with the radiation from the optically thick parts of the accretion disc, while the "hard" component of radiation is identified with the radiation from the neutron star's surface and mostly from the inner parts of the disc, most closely to the magnetosphere of the neutron star. The "hard" component varies considerably with time and therefore quite probably may account for the process of formation and destruction of large-sclale helicity structures in the boundary layer of the disc.

Both in the cases of accretion discs around black holes and around neutron stars one and the same problem arises: is the transition from "high" to "low" state (or vice versa) connected with a transition from chaotical to structurized turbulent motion? Unfortunately this problem in astrophysics has not yet been resolved due to serious obstacles and inadequate basic assumptions in the theory. However, if these assumptions are properly corrected and various new mathematical methods are implied, it will be possible to give a consistent explanation to this interesting phenomena within the framework of self-organization theory.

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Самоорганизационна теория и самоорганизационни проблеми на теорията на магнитохидродинамичните акреционни дискове по отношение на процеса на формиране на едромащабни турбулентни вихрови структури в акреционни дискове около черни дупки и неутронни звезди

Богдан Димитров

(Резюме)

Настоящата работа има за цел да формулира основните принципи за приложение на новия самоорганизационен (синергетичен) подход в теорията на магнитохидродинамичните акреционни дискове в астрофизиката. На основата на критичен анализ от гледна точка на самоорганизационната теория в настоящата работа е доказано, че по-нататъшното теоретично изследване на процеса на едромащабно вихрово структурообразуване се нуждае от значително модифициране на повечето от основните предположения в теорията на магнитохидродинамичните акреционни дискове.

Представени са и някои наблюдателни факти от астрофизиката на черните дупки и неутронните звезди в потвърждение на тезата за предполагаемото съществуване на такива структури в акреционни дискове около споменатите компактни астрофизични обекти.

Настоящата работа би могла да послужи като отправна точка за обширно теоретично изследване на турбулентното магнитохидродинамично вихрово структурообразуване.